Communications to SIMAI Congress,
ISSN 1827-9015, Vol. 1 (2006)
DOI: 10.1685/CSC06066

The application of a boundary integral equation solution method in parameter extraction for time-domain transmission line simulation

F. Delfino, R. Procopio and M. Rossi
Dipartimento di Ingegneria Elettrica, Università degli studi di Genova
Via Opera Pia 11a, I-16145 Genova, Italy. *E-mail: mansueto@die.unige.it*

Abstract

A large variety of problems in the field of *electro-magnetic compatibility* can be reduced to the evaluation of the response of a finite length line to an external electromagnetic field. This problem is mainly faced via the transmission line approximation, but the simplicity of this model and its relatively easy application both in frequency and time domain is counterbalanced by the underlining simplifying (and, in several cases, quite rude) assumptions. To overcome these assumptions, a more sophisticated mathematical model has to be considered. This latter, in the frequency domain is expressed, for a line of N conductors lying in the z direction, by the well-known integro-differential equation [1]:

$$\left[\frac{\mathrm{d}^2}{\mathrm{dz}^2}+\mathrm{k}^2\right]\sum_{\mathrm{i}=1}^{\mathrm{N}}\int_{-\mathrm{L}}^{\mathrm{L}}\mathrm{I}_\mathrm{i}(\mathrm{z}')\mathrm{g}_{\mathrm{ij}}(\mathrm{z}-\mathrm{z}')\,\mathrm{dz}'=-\mathrm{j}\frac{4\pi\mathrm{k}}{\xi_0}\mathrm{E}_{\mathrm{z\,inc,j}}, \tag{1}$$

linking the *z* component of the incident electric field on each conductor *j*, namely $E_{z\ inc,j}$, to the currents I_i flowing in the conductors; $g_{ij}(z\text{-}z')$ are suitable Green functions, depending on the problem geometry, *k* is the wave number and ξ_0 is the free space impedance.
An efficient technique to solve this problem with better performance with respect to, for instance, classical Method of Moments (MoM) implementations, has been proposed in [2] and extended in [1] and [3] for multiconductor open circuited and loaded lines. In this latter cases, the considered loads were simple current injections at line ends, allowing to formulate and solve the model entirely in frequency domain. But when more realistic and complex problems have to be considered, the use of time domain simulators becomes mandatory: classical examples are, in the context of electric power system, lightning induced overvoltages on power lines or on electric traction contact lines, and, in electronic circuit analysis, noise and potentially dangerous voltages caused on interconnects and busses by partial discharges. In these situations, only the use of power system oriented or electronic circuit oriented time domain simulators allow to take into account networks, electric stations or feeding circuit connected to the line. The aforementioned method has then to be integrated with this latter kind of tools. To this end, rather than perform a direct coupling between them, we developed a procedure to obtain from the proposed technique a line equivalent representation of the kind:

$$\begin{bmatrix}[\mathrm{V}_1]\\ [\mathrm{V}_2]\end{bmatrix}=\begin{bmatrix}[\mathrm{Z}_{11}] & [\mathrm{Z}_{12}]\\ [\mathrm{Z}_{12}] & [\mathrm{Z}_{22}]\end{bmatrix}\begin{bmatrix}[\mathrm{I}_1]\\ [\mathrm{I}_2]\end{bmatrix}+\begin{bmatrix}[\mathrm{V}_1^{\mathrm{E}}]\\ [\mathrm{V}_2^{\mathrm{E}}]\end{bmatrix}+\begin{bmatrix}[\mathrm{V}_1^{\mathrm{S}}]\\ [\mathrm{V}_2^{\mathrm{S}}]\end{bmatrix}, \tag{2}$$

where $[I_1]$ and $[I_2]$ are the vectors of the injected current at the ends, $[V_1]$ and $[V_2]$ are the ends voltages, $[Z_{ij}]$ are impedance submatrices, $[V_1^E]$, $[V_2^E]$ are the open circuited voltages directly due to the incident electric field (obtained directly integrating the vertical component of this latter in correspondence of the line ends) and $[V_1^S]$, $[V_2^S]$ are the open circuited voltages due the induced

Licensed under the Creative Commons Attribution Noncommercial No Derivatives

currents (i.e. the *scattered voltages*). The choice of this representation naturally leads to a three-step procedure:

1. evaluation of the open-circuit voltages in the frequency domain;
2. evaluation of the impedance matrix in the frequency domain;
3. translation of the quantities evaluated at step (1) and (2) into a form suitable for time domain analysis.

The third step, for the open circuit voltages, is carried out by means of the vector fitting technique (VF) developed in [4], whereas to accomplish the second one the simplest way is to resort to the classical model derived by the modal analysis [5]. As far as the first and most important step is concerned, our field-to-line coupling frequency domain method has been used as follows: starting from equation (1), with a suitable normalization and adopting a vector notation, we obtain:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\varsigma^2}+\hat{\mathrm{k}}^2\right]\int_{-1}^{1}\left[\mathrm{g}(\varsigma-\varsigma')\right]\left[\mathrm{I}(\varsigma')\right]\mathrm{d}\varsigma' = 4\left[\mathrm{F}(\varsigma)\right], \tag{3}$$

where $\hat{k}=k\cdot L$, $\varsigma=z/L$ and $\left[F(\varsigma)\right]=\frac{\pi k}{j\xi_0}\left[E_{z,inc,1}(z/L)\ldots E_{z,inc,N}(z/L)\right]^{\mathrm{T}}$.

It can be shown [1] that system (3) is of singular kernel type, since the Green functions g_{ii} exhibit a logarithmic singularity as their arguments approach zero. As a consequence, a proper expansion for the unknowns is [6]:

$$\left[\mathrm{I}(\varsigma)\right]=\left(1-\varsigma^2\right)^{\frac{1}{2}}\sum_{n=0}^{+\infty}\frac{[\mathrm{s_n}]}{\pi\alpha_\mathrm{n}}\mathrm{C}_\mathrm{n}^1(-\varsigma), \tag{4}$$

where C_n^1 is the nth Gegenbauer polynomial of kind 1, $\alpha_n = j^n(n+1)/\pi$ is a constant introduced for convenience, and $[s_n]$ are vectors of unknown coefficients. Using this expansion and projecting the resulting equation on $C_q^1(\varsigma)$, the following equation is obtained:

$$\sum_{n=0}^{+\infty}\frac{1}{4\pi\alpha_\mathrm{n}}\left\{\int_{-1}^{1}\left[\frac{\mathrm{d}^2}{\mathrm{d}\varsigma^2}+\hat{\mathrm{k}}^2\right](1-\varsigma^2)^{\frac{1}{2}}\cdot\int_{-1}^{1}\left(1-\varsigma'^2\right)^{\frac{1}{2}}\left[\mathrm{g}(\varsigma-\varsigma')\right]\mathrm{C}_\mathrm{n}^1(-\varsigma')\mathrm{C}_\mathrm{q}^1(\varsigma)\mathrm{d}\varsigma'\mathrm{d}\varsigma\right\}[\mathrm{s_n}]=$$

$$=\int_{-1}^{1}(1-\varsigma^2)^{\frac{1}{2}}\left[\mathrm{F}(\varsigma)\right]\mathrm{C}_\mathrm{q}^1(\varsigma)\mathrm{d}\varsigma, \tag{5}$$

or, in a more compact form:

$$\sum_{n=0}^{\infty}\left[\mathrm{B_{qn}}\right][\mathrm{s_n}]=\left[\mathrm{F_q}\right]. \tag{6}$$

It can be shown (see [1-2], where a slightly different but equivalent approach has been followed) that the matrix $\left[B_{qn}\right]$ can be evaluated by means of the following series:

$$\left[\mathrm{B_{qm}}\right]=\mathrm{c_{mq}}\sum_{\mathrm{r}=-\infty}^{+\infty}\left(\hat{\mathrm{k}}^2-\frac{\pi^2\mathrm{r}^2}{4}\right)\frac{\mathrm{J_{m+1}}(\pi\mathrm{r}/2)\mathrm{J_{q+1}}(\pi\mathrm{r}/2)}{(\pi\mathrm{r}/2)^2}[\mathrm{G_r}], \tag{7}$$

where:

$$[\mathrm{G_r}]=\frac{1}{4}\int_{-1}^{1}\left[\mathrm{g}(\varsigma)\right]\mathrm{e}^{-\mathrm{j}\frac{\pi\mathrm{r}}{2}\varsigma}\mathrm{d}\varsigma, \tag{8}$$

$c_{qm}=2\pi\alpha_m\alpha_q$ and J_m are first-order Bessel functions.

The inversion of system (6) enables us to find the lightning induced currents on the open-circuited line. In principle, once the currents have been computed, the overvoltages along the line and in particular at line ends can be found via integration: this is generally a time-consuming step. One of the most remarkable aspects of this method is that, with suitable manipulations [7], these integrations can be completely avoided: the line end overvoltages can be expressed directly in terms

of the incident field coefficients (namely, r.h.s. of system (6)) via parameters that can be computed and stored over a wide range of frequencies. Thus, only one frequency analysis is needed in order to find these parameters and then the response to different incident electric fields can be computed.
Once the open-circuit overvoltages have been calculated in the frequency domain, the third step of the procedure remains to be accomplished by means of the vector fitting: this method allows to approximate a generic transfer function *H(s)* in a sum of simple fractional functions of the kind:

$$H(s) \approx \sum_{n=1}^{N} \frac{c_n}{s-a_n} + d + s \cdot e, \tag{9}$$

being *s* the variable in the complex plane, a_n real or conjugate complex poles, c_n the residuals, *e* a constant equal to zero for *proper functions*, *d* a constant, which vanishes for *strictly proper functions*. Basically, the method consists of an iterative procedure, which uses the least mean square method to minimize the difference between *H(s)* and its approximation over a range of sampled points.

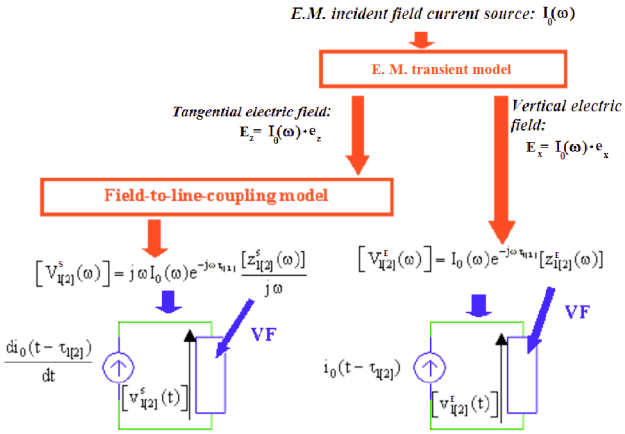


Fig. 1. Open-circuit voltages representation.

This feature is used in order to evaluate directly in time domain the open-circuit overvoltages as follows: with reference to figure 1, the scattered open-circuit overvoltages appearing in equation (2) are due to the components of the incident electric field tangential to the line, namely E_t, while the direct contributions $[V_1^E]$, $[V_2^E]$ are due to the vertical electric field E_x; both these field can be related to a transient (e.g. lightning or partial discharge) and are linear with the current generating it (e.g. the lightning channel base current or the partial discharge current); therefore, in a generic point *P(x, y, z)* they can be expressed as the product of $I_0(\omega)$ and the functions $e_t(\cdot)$ and $e_x(\cdot)$ depending on the distance between *P* and the transient source and on the transient model:

$$E_t = I_0(\omega) \cdot e_t,\ E_x = I_0(\omega) \cdot e_x, \tag{10}$$

The same property of linearity characterizes also the relationships $[V_{1(2)}^E]$-$I_0(\omega)$ and $[V_{1(2)}^S]$-$I_0(\omega)$:

$$\left[V^E_{1[2]}(\omega)\right] = I_0(\omega) e^{-j\omega\tau_{1[2]}} [z^E_{1[2]}(\omega)], \left[V^S_{1[2]}(\omega)\right] = j\omega I_0(\omega) e^{-j\omega\tau_{1[2]}} \frac{[z^S_{1[2]}(\omega)]}{j\omega}, \tag{13}$$

Practically, both the contributions to the open-circuit overvoltages can be written without loss of generality as the product among $I_0(\omega)$, suitable "delay factors" of the kind $e^{-j\omega\tau_{1[2]}}$ and the functions $z_{1[2]}^E(\omega)$ and $z_{1[2]}^S(\omega)$. The delay factors take into account the time required for the field wave to propagate from source point to the line ends, while the functions of the frequency $z_{1[2]}^E(\cdot)$ and $z_{1[2]}^S(\cdot)$ can be reconstructed as "impedances" by means of the VF technique.
As an example of application of the outlined procedure, let us consider a typical module of a railway transportation system powered at 750 V (see figure 2). The voltage is fed by two ac/dc stations and distributed by a two-conductors Overhead Transmission Lines (OTL). The module length is 400m, while his height is 6.5 m. The system is equipped by surge arresters at both ends of each feeding cable.

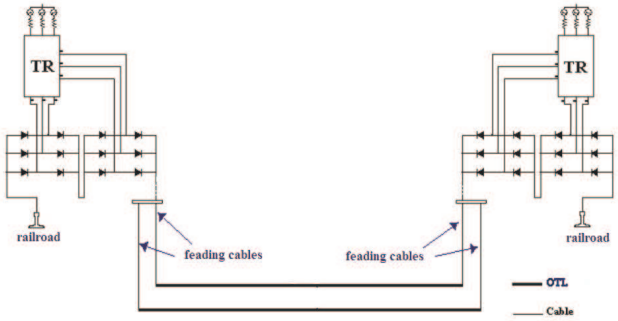


Fig. 2 – A typical module of a light railway transportation system network.

Cables, surge arresters and feeding station equipments have been modeled in a commercial power system oriented circuit simulator, together with the line circuit model equivalent to (2). A lightning stroke hitting the ground 50 m far from the line, in correspondence of its middle section, has been considered. Classical models for channel base current and lightning discharge have been used (see [7] for details). The overvoltages calculated at one line end and at one of the electric station terminals are reported in figure 3. Results like these give important insight, for instance, in protection system design, showing that the approach can constitute a useful prediction tool in a large variety of EMC problems.

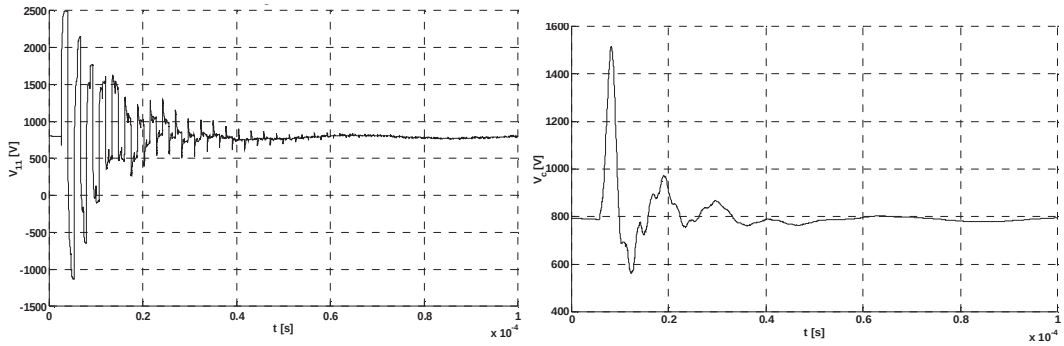


Fig. 3. Overvoltages at one of the line ends (left) and at one of the electric station terminals (right).

References

[1] F. Delfino, M. Nervi, R. Procopio, M. Rossi, A full-Maxwell algorithm for the field-to-multiconductor line-coupling problem, COMPEL, 22 (2003), 789-805.

[2] F. Delfino, R. Procopio, M. Rossi, A new method for the solution of dual integral equations systems occurring in electromagnetism, Proc. of the SIMAI conference (2002), Chia Laguna.

[3] M. Rossi, Full-Maxwell analysis of loaded multiconductor lines, Proc. of the PIERS symposium (2004), Pisa.

[4] B. Gustavsen and A. Semlyen, Rational approximation of frequency domain responses by vector fitting, IEEE Trans. on Power Delivery, 14 (1999), 1052-1061.

[5] C. R. Paul, Analysis of Multiconductor Transmission Lines, New York: Wiley, 1994.

[6] Yu. V. Shestopalov, Yu. G. Smirnov, E. V. Chernokozhin, Logarithmic Integral Equations in Electromagnetics, VSP Zeist, Utrecht, (2000).

[7] F. Delfino, R. Procopio, M. Rossi, Characterization of a railways system overhead transmission line excited by a lightning electromagnetic field, Proc. of the ELECTROCOMP conferece (2005) Orlando (FL), 631-641.