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Thermogalvanomagnetic effects in the vortex field of type-II superconductor

Bogdan T. Maruszewski
Poznan University of Technology, Institute of Applied Mechanics
Ul. Piotrowo 3, 60-965 Poznan, Poland
e-mail: bogdan.maruszewski@put.poznan.pl

The motion of vortices in a type-II superconductor is accompanied by a heat flux coming from the vortices themselves [1]. It leads to such thermogalvanomagnetic effects like the Nernst, Ettingshausen and Righi-Leduc ones. Moreover, besides linear thermoelectric Seebeck's and Peltier's effects also the Hall effect occurs. That situation seems to be very interesting because takes place not for common electric conductivity processes but during diffusion and/or creep of magnetic vortices in superconductors. It is known that each vortex line carries a quantum of magnetic field and around it a supercurrent flows. But inside the vortex core a normal current exists. Therefore, the above kinetic linear and nonlinear effects are possible in the vortex array.

One of the most efficient way to model those processes with such effects is unconventional nonequilibrium and extended thermodynamics [2-4] because all physical processes run in low temperatures. Therefore their relaxation properties can be modeled and described.

The unconventional thermodynamical model presented in the paper consists of [4]

- set of independent variables (vector of state):

$$C=\left\{\varepsilon_{ij},\varphi,A_i,T,T_{,i},c,c_{,i},\psi,\psi^*,\psi_{,i},\psi^*_{,i},q_i,j_i^c,j_i^S\right\}, \tag{1}$$

where ε_{ij} is the strain tensor of the vortex field, φ and A_i are electromagnetic potentials, T is temperature, c is the concentration of magnetic vortices, ψ is the wave function of Cooper's pairs, q_i is the heat flux, j_i^c is the vortex flux, j_i^S is the supercurrent (flux of superelectrons – Cooper's pairs);

- balances:

$$\begin{gathered}\rho\dot{c}+j^c_{k,k}=0,\\ \rho\dot{v}_k-\sigma_{jk,j}-\in_{kij} j_iB_j-f_k=0,\\ \in_{ijk}\sigma_{jk}=0,\\ \rho\dot{U}-\sigma_{ji}v_{i,j}+q_{k,k}-j_i\xi_i-\rho r=0,\end{gathered} \tag{2}$$

where ρ is the mass density of vortices, v_k is the velocity of a vortex field point, σ_{ji} its stress tensor, B_i is the magnetic induction of the applied magnetic field, f_k is the body force, U is the internal energy density, ξ_i is the electric field of the normal phase in the moving frame and r is the heat source;

- electromagnetic equations and relations:

$$\begin{aligned}&\in_{ijk}E_{k,j}+\frac{\partial B_i}{\partial t}=0, && \xi_i=E_i+\in_{ijk}v_jB_k,\\ &\in_{ijk}H_{k,j}-j_i=0, && j_i=j_i^N+j_i^S,\\ &B_{k,k}=0, && E_k=-\varphi_{,k}-\frac{\partial A_k}{\partial t},\\ &D_{k,k}=0, \quad B_k=\in_{ijk}A_{j,i}, && D_k=\in E_k, \quad B_k=\mu_0H_k,\end{aligned} \tag{3}$$

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where H_k is the magnetic field intensity, $\in$ is the permittivity, μ_0 is permeability of vacuum, j_i^N is the normal current and j_i is the total current;

- evolution equations:

$$\begin{aligned} &\overset{*}{q}_k - Q_k(C) = 0, \\ &\dot{\psi} - \Psi(C) = 0, \\ &\dot{\psi}^* - \Psi^*(C) = 0, \\ &\overset{*}{j}{}_k^c - j_k^c(C) = 0, \\ &\overset{*}{j}{}_k^S - j_k^S(C) = 0, \end{aligned} \tag{4}$$

where superimposed asterisk denotes the Zaremba-Jaumann derivative;

- balance of superelectrons:

$$\frac{\partial n^S}{\partial t} + j_{k,k}^S = N^S(C), \tag{5}$$

$$j_{k,k}^S - N^S(C) = \left(\psi^* \psi_{,k} + \psi \psi_{,k}^*\right) - \left[\psi^* \Psi(C) + \psi \Psi^*(C)\right],$$

where $N^S(C)$ is the source-like term of superelectrons and n^S is the number density of Cooper's pairs;

- entropy inequality:

$$\rho \dot{S} + \Phi_{k,k} - \frac{\rho r}{T} \geq 0, \tag{6}$$

where S is the entropy and Φ_k is its flux;

- set of dependent variables (constitutive functions)

$$Z = \left\{\sigma_{ij}, \mu^c, U, Q_k, \Psi, \Psi^*, J_k^c, J_k^S, S, \Phi_k\right\}, \tag{7}$$

A detailed analysis of the entropy inequality leads us to the following residual inequality which stands for the kinetic part of the modeled and described interaction among the elastic, thermal, diffusion and electromagnetic fields in the vortex array:

$$\begin{aligned} &-\frac{1}{T} q_k T_{,k} - h_c j_k^c c_{,k} + j_i^N \xi_i - \rho \frac{\partial F}{\partial q_i} \dot{q}_i - \rho \frac{\partial F}{\partial j_i^c} \dot{j}_i^c - \rho \frac{\partial F}{\partial j_i^S} \dot{j}_i^S - \\ &- \left[\rho \frac{\partial F}{\partial \psi} - \left(\rho \frac{\partial F}{\partial \psi_{,k}}\right)_{,k}\right] \frac{\partial \psi}{\partial t} - \left[\rho \frac{\partial F}{\partial \psi^*} - \left(\rho \frac{\partial F}{\partial \psi_{,k}}\right)_{,k}\right] \frac{\partial \psi^*}{\partial t} \geq 0, \end{aligned} \tag{8}$$

where F denotes the free energy, a thermodynamical potential which defines laws of state, affinities and with the entropy flux also kinetic relations [4],[5].

That inequality can be classically presented in the form:

$$J_\alpha X_\alpha \geq 0, \qquad J_\alpha = \ell^{\alpha\beta} X_\beta, \tag{9}$$

with fluxes linear in forces. The Onsager-Casimir relations read

$$\ell^{\alpha\beta} = \ell^{\beta\alpha}. \tag{10}$$

To reach our aim of the paper we formulate J_α matrix and X_α matrix as follows:

$$J_\alpha = \begin{Bmatrix} q_k \\ j_k^c \\ j_k^N \\ \dot{q}_k \\ \dot{j}_k^c \\ \dot{j}_k^S \\ \dot{\psi} \\ \dot{\psi}^* \end{Bmatrix}, \qquad X_\beta = \begin{Bmatrix} -\dfrac{1}{T}T_{,k} \\ -h_c c_{,k} \\ \xi_i \\ -\rho\dfrac{\partial F}{\partial q_i} \\ -\rho\dfrac{\partial F}{\partial j_i^c} \\ -\rho\dfrac{\partial F}{\partial j_i^S} \\ -\left[\rho\dfrac{\partial F}{\partial \psi} - \left(\rho\dfrac{\partial F}{\partial \psi_{,k}}\right)_{,k}\right] \\ -\left[\rho\dfrac{\partial F}{\partial \psi^*} - \left(\rho\dfrac{\partial F}{\partial \psi^*_{,k}}\right)_{,k}\right] \end{Bmatrix}. \tag{11}$$

and the phenomenological coefficient matrix in the following way:

$$\ell^{\alpha\beta} = \begin{Bmatrix} \ell^{11} & \ell^{12} & \ell^{13} & 0 & 0 & 0 & 0 & 0 \\ \ell^{21} & \ell^{22} & \ell^{23} & 0 & 0 & 0 & 0 & 0 \\ \ell^{31} & \ell^{32} & \ell^{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ell^{44} & \ell^{45} & \ell^{46} & 0 & 0 \\ 0 & 0 & 0 & \ell^{54} & \ell^{55} & \ell^{56} & 0 & 0 \\ 0 & 0 & 0 & \ell^{64} & \ell^{65} & \ell^{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ell^{77} & \ell^{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & \ell^{87} & \ell^{88} \end{Bmatrix}. \tag{12}$$

The basic thermogalvanomagnetic effects and effects which include relaxation features of the considered processes in the first approximation can be described if the phenomenological coefficients are assumed in the following form:

$$\ell_{ij}^{\alpha\beta} = \ell_{ij}^{\alpha\beta(0)} + \ell_{ijk}^{\alpha\beta(1)} H_k. \tag{13}$$

After labor consuming calculations the final forms of the expected kinetic relations both without and with relaxation properties (for the sake of simplicity and easy interpretation we present them in the isotropic form assuming that $\ell_{kj}^{\alpha\beta(0)} = \ell^{\alpha\beta(0)}\delta_{kj}$, $\ell_{kjl}^{\alpha\beta(1)} = \ell^{\alpha\beta(1)} \in_{kjl}$) :

$$\mathbf{q} = -\kappa\nabla T + \frac{1}{T}\ell\nabla T\times\mathbf{H} - h_c\kappa^2\nabla c + h_c K^c\nabla c\times\mathbf{H} + \kappa^e\xi + N\xi\times\mathbf{H} \tag{14}$$

$$\mathbf{j}^c = -\frac{1}{T}\kappa^c\nabla T + \frac{1}{T}K^c\nabla T\times\mathbf{H} - \rho D\nabla c + Mh_c\nabla c\times\mathbf{H} + \Sigma^c\xi + \Gamma^c\xi\times\mathbf{H} \tag{15}$$

$$\mathbf{j}^N = -\frac{1}{T}\kappa^e\nabla T + \frac{1}{T}N\nabla T\times\mathbf{H} - h_c\Sigma^c\nabla c + \rho D\nabla c + h_c\Gamma^c\nabla c\times\mathbf{H} + \sigma\xi + R\xi\times\mathbf{H} \tag{16}$$

$$\tau^q \dot{\mathbf{q}} = \kappa \nabla T - \frac{1}{T}\ell \nabla T \times \mathrm{H} + h_c \kappa^2 \nabla c - h_c K^c \nabla c \times \mathbf{H} + \kappa^e \xi - N\xi \times \mathbf{H} - \\ - \mathbf{q} - D^c \mathbf{j}^c - D^S \mathbf{j}^S \tag{17}$$

$$\tau^c \dot{\mathbf{j}}^c = \frac{1}{T}\kappa^c \nabla T - \frac{1}{T} K^c \nabla T \times \mathbf{H} + \rho D \nabla c - M h_c \nabla c \times \mathbf{H} + \Sigma^c \xi - \Gamma^c \xi \times \mathbf{H} - \\ - D^q \mathbf{q} - \mathbf{j}^c - D^{Sq} \mathbf{j}^S \tag{18}$$

$$\tau^S \dot{\mathbf{j}}^S = P^T T \nabla T - \frac{1}{T} R^T \nabla T \times \mathbf{H} + P^c \nabla c - h_c R^c \nabla c \times \mathbf{H} + \frac{1}{\mu_0 \lambda_0^2}\xi - R^e \xi \times \mathbf{H} - \\ - D^{qS} \mathbf{q} - \mathbf{j}^S - D^{cS} \mathbf{j}^c . \tag{19}$$

In (14) which is the extended Fourier law we recognize κ as the heat conductivity coefficient, ℓ as Righi-Leduc coefficient, h_c as thermodiffusive constant, κ^c as the Dufour coefficient, K^c as the Peltier constant, N as the Nernst-Ettingshausen coefficient. In (15) which is the extended Fick law we recognize D as the diffusion constant, M as the magnetodiffusive coefficient, Σ^c as electrodiffusive coefficient, Γ^c as the electromagnetodiffusive coefficient. In (16) which is the extended Ohm law we recognize σ as the electrical conductivity and R as the Hall coefficient. In (17) which is the Vernotte-Cattaneo relation we recognize τ^q as the thermal relaxation time, D^c the extended thermodiffusive coefficient and D^S as the superthermal coefficient. In (18) which is the Fick-Nonnenmacher extended law we recognize τ^c as the diffusion relaxation time, D^q as the thermodiffusive kinetic coefficient and D^{Sq} as the superdiffusive coefficient. In (19) which is the second London equation we recognize τ^S as the vortex relaxation time, P^T as the superthermal kinetic coefficient, R^T superthermomagnetic coefficient, P^c as the superdiffusive kinetic coefficient, R^c as the superdiffusiomagnetic kinetic coefficient, λ_0 as the London penetration depth, R^e as the electromagnetic kinetic coefficient, D^{qS} as the superdiffusive kinetic coefficient.

In conclusion remark that besides of commonly known thermogalvanomagnetic effects described by relations (14) and (16) some new effects can be observed because of diffusion of vortices (see (15)). Moreover, it seems that a relaxation and possibility of waves in the vortex field is possible (see (17) and (19)).

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