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An Integrated Approach for the Segmentation of Color Images

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Abstract

In this paper we propose an integrated procedure for the segmentation of decay regions from color images of monuments. The integrated procedure consists in combining the numerical techniques of anisotropic diffusion and fast marching. The anisotropic diffusion has been introduced to limit the smoothing at zones of high gradient in the images. Then we apply a numerical algorithm based on the theory of interface evolution to the images obtained after the diffusion process. The study case concerns the impressive remains of the city of Aosta (Italy).

Keywords: `Numerical algorithm, segmentation, color coordinates, color image`

1. Introduction

In the Cultural Heritage the use of appropriate image analysis techniques can contribute to analyzing the decay of ancient monuments. In particular the application of an image segmentation strategy to color images of stony materials can be an important step to extract decay regions characterized by chromatic alteration.
The decay region analysis on ancient monuments is a fundamental problem related to the state of preservation of historical buildings. On the wall facades of the monuments there are different types of decay such as oxidation, sediment and/or cavities which are recognizable by the feature of the color. The acquired images, related to the facades of a monument, show, for example, reddish stains due to oxidation, whitish ones due to the sediment of saline efflorescence or generally dark / black ones due to lack of material as in cavities. So in the images the decay areas are characterized by pixels of different color [1].

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In our application we face the problem to extract decay regions from color images by using the fast marching method. This method solves the eikonal equation, whose solution represents the boundary of the Region Of Interest (ROI) [2,3,4].
This approach allows us to use a segmentation technique, that works in a local way, that is it extracts from the image, only the region under study.
The numerical model, based on finite difference technique and fast marching method, approximates the evolution of a curve, that, starting from a seed point located in the region of interest, expands by incorporating step by step an increasing number of pixels up finding the contour of the ROI [5,6].
But then the decay areas show colored pixels, which are not distributed in a homogeneous way. For example, an oxidation stain, in an image, can contain red pixels or red-yellow pixels together with very dark pixels due to dust. This may make the extraction of the decay from color images very difficult.
For this reason we used the anisotropic diffusion to make the decay regions more uniform and therefore simplify the next process of segmentation. The basic idea is to smooth the regions and to reduce the smoothing effect near edges [7]. The related numerical algorithm generates a sequence of smoother images obtained while the evolution time goes by (space scale).
In this paper we propose an extension of anisotropic diffusion to color images. In this case the vector components are diffused independently of one another, but the diffusion coefficient is defined fusing the three components of the image [8]. The developed procedure constituted by anisotropic diffusion and curve evolution to segment decay regions has been applied to extract degraded areas from the images of the Roman impressive remains in the city of Aosta [9].
Every pixel of the images has been analyzed in $L^*a^*b^*$ color components. CIEL*a*b*, abbreviated CIELAB, color space has been defined by the CIE (Commission International de l'Eclairage) as a uniform color space [10,11].
For our application, we have used color images taken from the Research Project between ITABC-CNR and the Superintendence of Aosta.
The paper is structured as follows. Section 2 describes the anisotropic diffusion, its extension to color images and the segmentation algorithm. Section 3 contains some experimental results. Some conclusions are drawn in Section 4.

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2. Diffusion Tensor and Color Image Segmentation

In image analysis it is fundamental to smooth the regions of the image preserving their boundaries.
To overcome this problem, Perona and Malik [7] proposed a nonlinear adaptive diffusion process, termed anisotropic diffusion, for gray levels images, described by the PDE (Partial Differential Equation) equation

$$\frac{\partial I(x,y,t)}{\partial t} = \nabla \cdot (g(|\nabla I|)\nabla I), \qquad I(x,y,0) = I_0(x,y) \tag{2.1}$$

where $g(|\nabla I|) > 0$ is a decreasing function of the gradient.
In this model the smoothing process is given by the value of $|\nabla I(x,y)|$. If it is high the diffusion is low and therefore the "exact" location of the edges is obtained. Instead, if $|\nabla I(x,y)|$ is low the diffusion leads to smooth still more around (x,y). In this paper we choose as diffusion coefficient function

$$g(|\nabla I|) = \frac{1}{1 + (\frac{|\nabla I|}{k})^2}$$

where the parameter k is a constant and acts as an "edge threshold". The choice of the k parameter is critical. If the value is too large, the result of the diffusion process will give a blurred image; whereas if the value of k is too small the result will be an image similar to the original one. For this reason we assume the parameter k based on the mean gradient magnitude of the whole image [12].
In order to extend this approach to the color images, let $\mathbf{I} : \Omega \to \mathbb{R}^n$ be an intensity image function with the domain $\Omega \subset \mathbb{R}^2$, that maps a point to n-vector $\mathbf{I}$. This definition clearly includes monochromatic or gray level images as in particular case ($n = 1$). A color image ($n = 3$) is constituted by the three monochrome components [10], that is it may be represented as a vector function $\mathbf{I} = (f_1(x,y), f_2(x,y), f_3(x,y))$.
In image analysis a color image, visualized by additive mixtures of the **RGB** (Red, Green, and Blue) primary colors, can be processed in different color spaces as **RGB**, **XYZ** or CIEL$^*a^*b^*$.
The application of nonlinear diffusion PDEs to color images $\mathbf{I}$ is based on a local vector geometry [8], in this case the natural extension of the equation (2.1) becomes

$$\frac{\partial \mathbf{I}}{\partial t} = div(g(\|G\|) \cdot \nabla \mathbf{I}) \tag{2.2}$$

where $\|G\|$ is a norm of a tensor defined as

$$G = \sum_{j=1}^{n} \nabla f_j \nabla f_j^T \tag{2.3}$$

which is a 2×2 symmetric and semi-positive-definite matrix, called by us *diffusion tensor*, and gives the local variation of the image.
In this way $\|G\|$ fuses the different components of the image. In fact, the components are diffused independently of one another, but they are linked by means of the norm of the tensor G.
In (2.3), each ∇f_j corresponds to the spatial gradient of the jth vector component of the color image $\mathbf{I}$.
Let be

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix}$$

where

$$g_{11} = (\frac{\partial f_1(x,y)}{\partial x})^2 + (\frac{\partial f_2(x,y)}{\partial x})^2 + (\frac{\partial f_3(x,y)}{\partial x})^2$$

$$g_{22} = (\frac{\partial f_1(x,y)}{\partial y})^2 + (\frac{\partial f_2(x,y)}{\partial y})^2 + (\frac{\partial f_3(x,y)}{\partial y})^2$$

$$g_{12} = \frac{\partial f_1(x,y)}{\partial x}\frac{\partial f_1(x,y)}{\partial y} + \frac{\partial f_2(x,y)}{\partial x}\frac{\partial f_2(x,y)}{\partial y} + \frac{\partial f_3(x,y)}{\partial x}\frac{\partial f_3(x,y)}{\partial y}$$

The elements of G define the local geometry related to the image discontinuities. The eigenvalues $\lambda_\pm$ of G define the local min/max vector-valued variations of color image $\mathbf{I}$ corresponding to spatial directions given by the orthogonal eigenvectors $\theta_\pm$. More specifically

$$\lambda_\pm = \frac{g_{11} + g_{22} \pm \sqrt{\Delta}}{2}$$

and

$$\theta_\pm = \begin{pmatrix} 2g_{12} \\ g_{22} - g_{11} \pm \sqrt{\Delta} \end{pmatrix}$$

where $\Delta = (g_{11} - g_{22})^2 + 4g_{12}^2$.
We observe that $\lambda_+ \geq \lambda_- \geq 0$ and for scalar images when $n = 1$ $\lambda_+ = \|\nabla I\|$ and $\theta_+ = \frac{\nabla I}{\|\nabla I\|}$. Moreover if $\lambda_+ = \lambda_- = 0$, the region of interest is flat and does not contain any edges or corners; while if $\lambda_+ >> \lambda_-$, there are many edges in the region.
To complete the analogy with (2.1) we have to introduce a metric for G. We adopt as tensor norm

$$\Theta = \sqrt{trace(G)} = \sqrt{\lambda_+ + \lambda_-}$$

that in natural way extends the gradient norm of color images. In fact in

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the case of gray level images (n=1) the tensor Θ reduces to $|\nabla I|$. More specifically we can define

$$\Theta = \sqrt{\sum_i |\nabla f_i|^2}$$

and $||G|| = \Theta$.
Moreover the coefficient function $g(||G||)$ in (2.2) is

$$g(||G||) = \frac{1}{1 + (\frac{||G||}{k})^2} \tag{2.4}$$

where the parameter k is based on mean tensor norm $||G||$ of the whole image.
Now a curve evolution approach is proposed integrated with the previous anisotropic model.
Let $\tilde{\mathbf{I}} : \Omega \to \mathbb{R}^n$ be the *diffused* intensity image function.
The goal of image segmentation is based on partitioning Ω in order to extract disjoint regions from the image $\tilde{\mathbf{I}}$ such that they cover Ω. The boundaries of these regions may be considered as curves belonging to a family whose time evolution is described by a level set equation [2,3]. The main advantages of using the level set is that complex shaped regions can be detected and handled implicitly [13].
As in [14] the governing equation of a monotonically advancing front in the image domain is given by a non linear eikonal equation

$$|\nabla T(x, y)| = \frac{1}{F(x, y)} \tag{2.5}$$

where $T(x, y)$ is the function representing the arrival time at which the curve crosses a given point at the time t, $\frac{1}{F(x,y)} = f(x, y)$ is the so called curve slowness [14].
In details the known term $F(x, y)$ is typically computed in image form and it represents the front speed point by point. So, known $F(x, y)$, the solution of differential equation is the minimum time $T(x, y)$, for each pixel, necessary for the front to arrive at the pixel location.
It is well known that the choice of the speed term in the fast marching method is a fundamental task and in this paper we propose as speed function $F(x, y) = \frac{1}{1+||G||}$
In the opinion of sector expert scientists the procedure of integration between anisotropic diffusion and curve evolution has the advantage solving the problem of non-uniform distribution of the colors in the ROI without smoothing decay boundaries.

2.1. *Numerical Approximation*

For the numerical approximation of the equation (2.2) we introduce the computational domain Ω obtained by subdividing the original domain into N x M points.
Let $P_{i,j} \equiv P(x_i, y_j)$, $i = 1, \ldots, N$, $j = 1, \ldots, M$ be a point in Ω and $\mathbf{I}_{i,j} \equiv (f_1(x_i, y_j), f_2(x_i, y_j), f_3(x_i, y_j))$ we introduce the half steps in the x,y directions and $P_{i+\frac{1}{2},j}$, $P_{i,j+\frac{1}{2}}$, $P_{i,j-\frac{1}{2}}$, $P_{i-\frac{1}{2},j}$ are the corresponding computational points.

The equation (2.2) may be written

$$\frac{\partial \mathbf{I}(x,y,t)}{\partial t} = \frac{\partial (c(x,y,t)\mathbf{I}_x)}{\partial x} + \frac{\partial (c(x,y,t)\mathbf{I}_y)}{\partial y}$$

where $c(x,y,t) = g(||G(x,y,t)||$ and in the point $P_{i,j}$ we define $c^k_{i,j} = g(||G||^k_{i,j})$.

Usually in an image $\Delta x = \Delta y = 1$ and so the numerical scheme is

$$\mathbf{I}^{k+1}_{i,j} = \mathbf{I}^k_{i,j} + \Delta t \left[c^k_{i+\frac{1}{2},j}(\mathbf{I}^k_{i+1,j} - \mathbf{I}^k_{i,j}) - c^k_{i-\frac{1}{2},j}(\mathbf{I}^k_{i,j} - \mathbf{I}^k_{i-1,j}) \right] + \Delta t \left[c^k_{i,j+\frac{1}{2}}(\mathbf{I}^k_{i,j+1} - \mathbf{I}^k_{i,j}) - c^k_{i,j-\frac{1}{2}}(\mathbf{I}^k_{i,j} - \mathbf{I}^k_{i,j-1}) \right] \tag{2.6}$$

where Δt is the integration time step.
Moreover let $T_{i,j}$ be the value of the function $T(x,y)$ in $P_{i,j}$. The following formula, based on an upwind approximation, numerically solves the eikonal equation (2.5)

$$[max(D^{-x}_{i,j}T, -D^{+x}_{i,j}T, 0)^2 + max(D^{-y}_{i,j}T, -D^{+y}_{i,j}T, 0)^2]^{\frac{1}{2}} = f_{i,j} \tag{2.7}$$

where $D^-_{i,j}$ and $D^+_{i,j}$ are the usual backward and forward finite difference operators, respectively. For example we have

$$D^{-x}_{i,j}T = \frac{T_{i,j} - T_{i-1,j}}{x_i - x_{i-1}}.$$

and analogously for the other operators. It is well known that this approximation is consistent and stable [2,3,4].
In order to solve the nonlinear equation (2.7), we have adopted the fast marching method because it seems more appropriate for image segmentation applications. The algorithm starts from an initial point, named seed

point. The front evolving from this point, propagates to the normal direction with a special speed $F(x, y)$ [6,13].
We underline that we have to calculate the arrival times $T_{i,j}$ in order for the front to cross the boundary point $P_{i,j}$ and the values of the function $F_{i,j}$ are known in every point of the computational domain. We can observe that in this algorithm each grid point depends only on the smaller adjacent values and thus we can build the solution in order to increase values of $T_{i,j}$.
Moreover the fast marching algorithm may be connected with Huyghen's principle and with the Dijkstra's method; the first asserts that the wavefront of a propagating wave at any instant produces an envelope of spherical waves emanating from every point on the wavefront at the prior instant, the second is a depth-search technique for computing shortest paths on a network.
Therefore the solution of the eikonal equation may be interpreted in the following way: circular wave fronts are drawn at each point on the boundary with the radius proportional to $f(x, y)$. The envelope of these wavefronts is used to construct a new set of points and the process is repeated until the limit of the eikonal solution is obtained.

3. Experimental Results

In this section we show the results obtained by applying the developed procedure, based on the integration of anisotropic diffusion model and curve evolution method to real images of the Roman Theatre and Arch of Augustus in the city of Aosta.
The Roman Theatre which dates back to the Augustan age (1st century A.D.) is a rare example of the Roman architecture of covered theatres. For our application, we have utilized color images taken from SIINDA (SIstema Innovativi di iNdagine e Diagnosi Assistita) ([15].
The acquired images have been geometrically and chromatically corrected by using a colorimetric target under controlled light conditions.
An example of the obtained results related to oxidation and sediment decays are shown in the figures 1 and 2, respectively.

The Arch of Augustus, dedicated to the Emperor Caesar Augustus, dates back to the same period when the town of Aosta was founded by the Romans in 25 B.C. It was built with local stone and is 11.50 meters tall.
The acquired images of this building have been geometrically corrected and ortho-rectified [9].
The contours related to stains of biological decay obtained by the application of the procedure are shown in figure 3; analogously the resulting contour of additional material of black color is shown in figure 4.

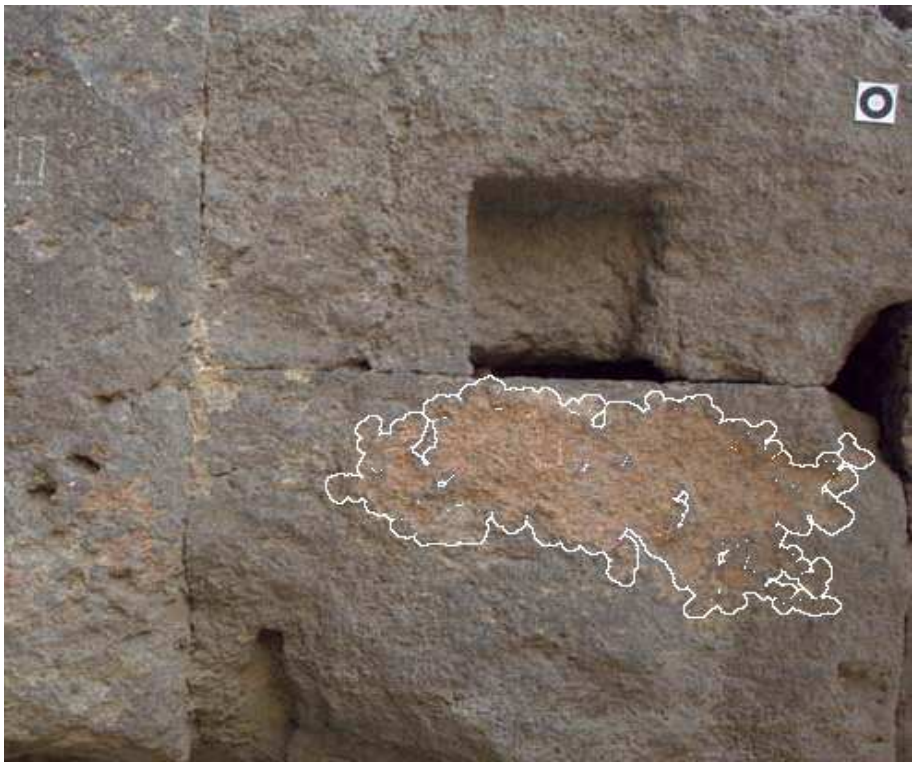

Fig. 1. Stain of oxidation

The prototype software has been implemented on a 2.4Ghz PC.

4. Conclusions

In this paper we have presented an integrated procedure of segmentation applied to color images, based on curve evolution method and an anisotropic diffusion extended to color images by local vector geometry. The tensor obtained fuses the different components of the image.
This procedure has been suggested by the type of the real images examined, which present decay regions constituted by pixels of different colors not uniformly distributed.

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Fig. 2. Stain of sediment

Fig. 3. Colored stains of biological decay

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Fig. 4. Black stain of material addition

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