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EKF application on estimating missile guidance signals

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Abstract

An Extended Kalman Filter (EKF) application has been developed to estimate relative position, relative velocity, Line Of Sight (LOS), LOS rate and target acceleration in the nonlinear missile-target relative kinematics. The used measures come from a strapdown seeker and an IMU. In order to improve the guidance performance, the setting of the filter against different types of target maneuvers has been studied. In all considered scenarios the EKF yields a remarkable improvement of miss distance values. The used approach and the filter set-up allow to include uncertainties such as misalignments, non synchronisms between sensors or seeker measures aberrations. The introduction of a supervision to adapt the setting to different identified target manoeuvres could be developed.

Keywords: `Target tracking, Extended Kalman Filter, Line of sight rate`

1. Introduction.

Several approaches have been proposed in the literature to deals with maneuvering targets (input estimation [1], noise covariances estimation [2], model switching [3] , IMMKF [4], etc.). Advanced guidance laws are used to improve terminal homing in the case of maneuvering target, however precise target states estimation are needed to apply these advanced guidance laws.

In this paper an Extended Kalman Filter to estimate the guidance signals essential to a missile having the objective of hitting a maneuvering target has been developed. Different kind of target maneuvers have been considered to test the filter performance. In the proposed solution the nonlinearities present in the interceptor vs target relative kinematics are con-

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sidered. The EKF filter introduces estimates of relative position, relative velocity and target acceleration in a 3D engagement between a pursuer and an evader. The estimates are obtained from the range, the range rate and the LOS direction cosines measurements. The signals used by the guidance algorithm are the (LOS) rate, when the Proportional Navigation law is used, and the target acceleration, when the Advanced Guidance law is used. As verified by numerical simulation, reduced miss distances are obtained when the EKF filter is applied.

The paper is organized as follows. In Section 2 the mathematical model of the nonlinear problem is presented. In Section 3 the application of the EKF algorithm to the problem is described. Section 4 contains the description of how the input to the filter can be obtained from the sensor measurements. In Section 5 the simulation results to evaluate the performance of the developed filter are described. Section 6 ends the paper with final conclusions.

2. Mathematical modeling

In order to apply the Kalman filter the following mathematical model is introduced.

State Equation
The state equation is a system of three vectorial equations:

$$\begin{aligned}
\dot{\underline{r}} &= \underline{v} \\
\dot{\underline{v}} &= \underline{n}_T - T^{-1}\underline{a}_M^B + \underline{g} + \underline{\omega}_a \\
\dot{\underline{n}}_T &= \underline{\omega}_j
\end{aligned}$$

where $\underline{r}$, $\underline{v}$ and $\underline{n}_T$ are respectively the relative position, the relative velocity and the target acceleration in the inertial reference frame, $\underline{a}_M^B$ is the missile acceleration in body coordinates, $\underline{g}$ is the gravity acceleration and T is the inertial to body transformation matrix. Finally, $\underline{\omega}_a$ and $\underline{\omega}_j$ are two independent stochastic processes which model the uncertainties in the considered problem. Acceleration $\underline{a}_M^B$ is estimated by the inertial measurement unit with some uncertainties, also matrix T, that is a function of the Euler angles, is estimated with some uncertainties. The stochastic process $\underline{\omega}_j$ is a Gaussian white noise which models the target acceleration as a random walk process. In fact, it can be shown [5] that a step maneuver $\underline{n}_T$, whose starting time is uniformly distributed over the flight time t_F has the same mean value and autocorrelation function as a linear system with transfer

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function $1/s$ driven by white noise with power spectral density $\Phi_s = n_T^2/t_F$. Therefore, the system state consists of nine components:

$$\underline{x} = \left[x_r \; y_r \; z_r \; v_x \; v_y \; v_z \; n_{T_x} \; n_{T_y} \; n_{T_z} \right]^T,$$

and the input is composed of interceptor and gravity accelerations:

$$\underline{u} = \begin{bmatrix} \underline{a}_M^B \\ \underline{g} \end{bmatrix}.$$

The considered state equation is linear and it can be written in matrix form as follows:

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B(t)\underline{u}(t) + B_1\underline{\omega}(t)$$

with:

$$A = \begin{bmatrix} 0 & I_3 & 0 \\ 0 & 0 & I_3 \\ 0 & 0 & 0 \end{bmatrix}, \quad B(t) = \begin{bmatrix} 0 & 0 \\ -T^{-1} & I_3 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ I_3 & 0 \\ 0 & I_3 \end{bmatrix},$$

and where:

$$\underline{\omega} = \left[\underline{\omega}_a^T \; \underline{\omega}_j^T \right]^T.$$

Measurement Equation
The considered measurements to estimate the state variables are the line of sight direction cosines, the range and the range rate, that are described by the following nonlinear equations:

$$\begin{aligned} LOS(t) &= \left[\frac{x_r(t)}{||r(t)||} \; \frac{y_r(t)}{||r(t)||} \; \frac{z_r(t)}{||r(t)||} \right]^T \\ ||r(t)|| &= \sqrt{x_r^2(k) + y_r^2(k) + z_r^2(k)} \\ ||\dot{r}(t)|| &= \frac{x_r(t)v_x(t) + y_r(t)v_y(t) + z_r(t)v_z(t)}{\sqrt{x_r^2(t) + y_r^2(t) + z_r^2(t)}} \end{aligned}$$

Then the measurement vector is:

$$\underline{z}(t) = \left[LOS(t)^T \; ||r(t)|| \; ||\dot{r}(t)|| \right]^T,$$

and the measure equation has the following form:

$$\underline{z}(t) = g\left[\underline{x}(t), t\right] + \underline{v}(t),$$

where $g(\cdot)$ is a non linear function, and $\underline{v}(t)$ is a white noise modeling the measurement errors. Therefore the dynamic system consists of a linear state

equation and a nonlinear measure equation:

$$\begin{array}{ll} \dot{\underline{x}}(t) = A\underline{x}(t) + B(t)\underline{u}(t) + B_1(t)\underline{\omega}(t) & \text{with} \quad \underline{\omega}(t) \backsim \mathcal{N}(0, Q(t)), \\ \underline{z}(t) = g\left[\underline{x}(t), t\right] + \underline{v}(t) & \text{with} \quad \underline{v}(t) \backsim \mathcal{N}(0, R(t)). \end{array}$$

Covariance matrices Q and R
The process noise covariance matrix Q is a block diagonal matrix which elements depend on the Inertial Measurement Unit variance Φ_{a_i} and on the power spectral density Φ_{s_i}. The measurement noise covariance matrix R is a diagonal matrix and is derived by the sensors devices.

3. EKF for signal guidance estimation

The Extended Kalman Filter has been applied to the previous dynamic system since the measurement equation is nonlinear. The linearisation and discretisation of the considered dynamic system produce:

$$\begin{array}{ll} \underline{x}(k+1) = \Gamma\underline{x}(k) + \Lambda(k)\underline{u}(k) + \underline{\omega}_d(k) & \text{with } \underline{\omega}_d(k) \backsim \mathcal{N}(0, Q_d(k)) \\ \underline{z}(k) = \overline{C}(k)\underline{x}(k) + \underline{v}(k) & \text{with } \underline{v}(k) \backsim \mathcal{N}(0, R(k)) \end{array}$$

where:

$$\begin{array}{l} \Gamma = e^{A\Delta t} \cong I + A\Delta t + \frac{1}{2}A^2\Delta t^2, \\ \Lambda(k) = \int_{t_k}^{t_{k+1}} e^{A(t-\tau)}B(\tau)d\tau, \end{array}$$

and:

$$\overline{C}(k) = \nabla g\left[x(k), k\right]_{x(k)=\hat{x}(k|k-1)}.$$

The following Kalman filter equations have been implemented:

$$\begin{array}{l} \hat{\underline{x}}(k|k) = \Gamma\hat{\underline{x}}(k-1|k-1) + \Lambda(k-1)\underline{u}(k-1) + K(k)\left[\underline{z}(k) - g\left(\hat{\underline{x}}(k|k-1), k\right)\right] \\ P(k|k-1) = \Gamma P(k|k-1)\Gamma^T + Q_d(k-1) \\ K(k) = P(k|k-1)\overline{C}^T(k)\left[\overline{C}(k)P(k|k-1)\overline{C}^T(k) + R(k)\right]^{-1} \\ P(k|k) = (I - K(k)\overline{C}(k))P(k|k-1). \end{array}$$

These equations take into account the linearized matrix $\overline{C}$ as well as the presence of the control input given by the missile and the gravity accelerations. By the state estimate and using the following trigonometric relationships:

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$$\lambda_o = \arctan\frac{LOS_y}{LOS_x} \qquad \lambda_v = \arctan\frac{LOS_z}{LOS_x}$$

$$\dot{\lambda}_o = \frac{\frac{L\dot{O}S_y LOS_x - LOS_y L\dot{O}S_x}{LOS_x^2}}{1+(\frac{LOS_y}{LOS_x})^2} \qquad \dot{\lambda}_v = \frac{\frac{L\dot{O}S_z LOS_x - LOS_z L\dot{O}S_x}{LOS_x^2}}{1+(\frac{LOS_z}{LOS_x})^2}$$

$$L\dot{O}S_x = \frac{v_x||r||^2 - x(\underline{r}\cdot\underline{v})}{||r||^3} \qquad L\dot{O}S_y = \frac{v_y||r||^2 - y(\underline{r}\cdot\underline{v})}{||r||^3}$$

it has been possible to derive the guidance signals that are used in the Proportional Navigation law or in the Advanced Guidance: the line of sight rates λ_o, λ_v and the target acceleration $\underline{n}_T$.

4. Sensor systems

The line of sight was obtained from the measurements through the reconstruction technique, using the boresight error ϵ, the antenna gimbal angles and the missile attitude θ_m.

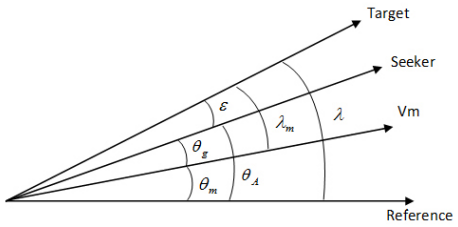

Fig. 1. Intercept geometry

Then the LOS direction cosines, used by the filter, are computed from the reconstructed line of sight using the following relationships:

$$\begin{aligned} LOS_x &= \cos\lambda_o^{ric}\cos(\tan^{-1}(\cos\lambda_o^{ric}\tan\lambda_v^{ric})) \\ LOS_y &= \sin\lambda_o^{ric}\cos(\tan^{-1}(\cos\lambda_o^{ric}\tan\lambda_v^{ric})) \\ LOS_z &= \sin(\tan^{-1}(\cos\lambda_o^{ric}\tan\lambda_v^{ric})) \end{aligned}$$

Preliminary simulations have been performed with the proposed model in order to estimate a compatible σ_{LOS} to give as input to the EKF filter. In addition, since the boresight error noise of the receiver decreases with decreasing range, a linear approximation for the estimate of σ_{LOS} has been proposed. In fact, more accurate approximation of this estimate does not produce significant improvement.

5. Simulation results

Different kinematic conditions, sensor parameters and EKF filter parameters have been considered in order to analyze the performance of the proposed solution on a large number of configurations.

State variable analysis - Single run
Consider the following input data:
Interceptor: $x_M = 0$, $y_M = 0$, $z_M = -400$, $v_M = 500$, $HE^{\theta_M} = HE^{\psi_M} = \pm 4°$
Target: $x_T = 5000$, $y_T = 100$, $z_T = -700$, $v_T = 300$, $\theta_T = 0$, $\psi_T = 180°$, $n_{T_y} = 3g$, $n_{T_z} = -2g$
Seeker and INS sensors std: $\sigma_\lambda = 0.016$, $\sigma_r = 10$, $\sigma_{\dot{r}} = 5$, $a_{M_x} = a_{M_y} = a_{M_z} = 0.001g$
EKF initialization: $\Phi_{s1} = \Phi_{s2} = \Phi_{s3} = \hat{n}_T^2/t_F$, $\hat{n}_T = 5$, $\Phi_{a1} = \Phi_{a2} = \Phi_{a3} = 0.001g$, $\hat{\sigma}_r = 10$, $\hat{\sigma}_{\dot{r}} = 5$, $\hat{\sigma}_{LOS_x} = \hat{\sigma}_{LOS_y} = \hat{\sigma}_{LOS_z} = 0.03$.
With these initial conditions, for a single run the estimated values converge to the true values of the states. Figure 2 shows that, after an initial transient, the filter gives good estimates of the system states.

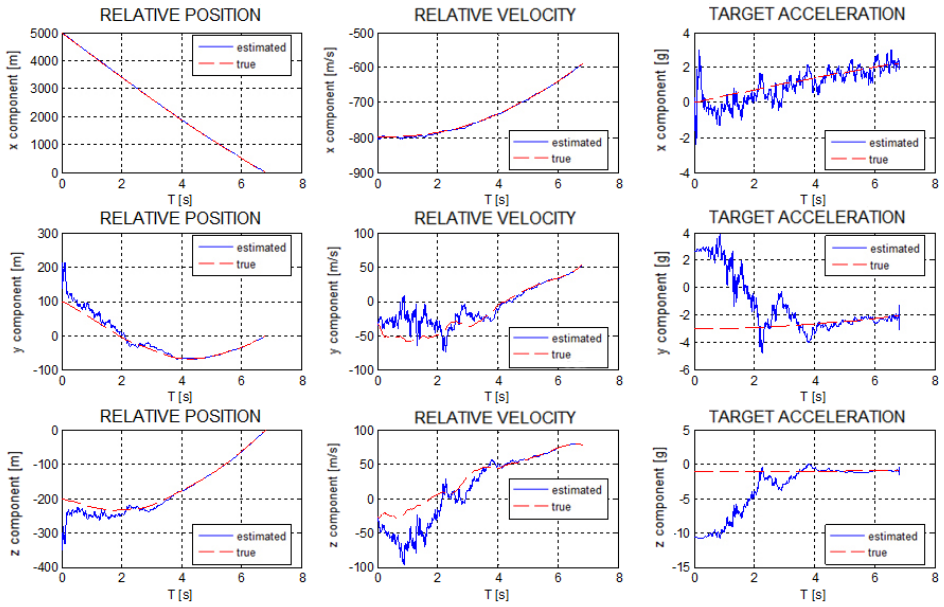

Fig. 2. Estimated and true state variables (dashed line: true value, solid line: estimated value).

Miss distance analysis - Multiple run
Three different types of target maneuvers have been considered to test the filter performance: the step maneuver, the weaving maneuver and the square

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wave maneuver. For all three conditions, the maneuver amplitude is n_T and the starting time t_i is a random variable uniformly distributed between 0 and the flight time t_F. Six different scenarios have been considered and for each of them 50 runs have been produced in order to study the resulting miss distances and the improvement given by the Kalman filter. The six scenarios have been chosen considering each maneuver first on a single plane and next on both planes. Table 1 shows miss distance parameters: the mean value, the standard deviation, the min and max values computed on the considered 50 runs in all six scenarios. In all of them the EKF filter leads to a remarkable improvement. In the case of a step target maneuver on both planes with amplitude established to $5g$ along the y plane and $-4g$ along the z plane, for instance, the average miss distance is $2.7m$ without EKF filter, $0.54m$ using the EKF filter and a constant matrix R, $0.17m$ using the EKF filter and a decreasing matrix R.

Table 1. Miss distance parameters.

		MEAN	STD	MIN	MAX
STEP MANEUVER	Without EKF	2.6125	2.0261	0.1405	7.9344
on single plane	With EKF and decreasing R	0.1628	0.1734	0.0131	0.8461
$n_{T_y} = 3g$, $n_{T_z} = 0g$	With EKF and decreasing R	0.4100	0.5919	0.0091	2.0739
STEP MANEUVER	Without EKF	2.7173	2.7215	0.0658	10.0649
on both planes	With EKF and decreasing R	0.1789	0.2267	0.0182	0.9480
$n_{T_y} = 5g$, $n_{T_z} = -4g$	With EKF and decreasing R	0.5463	0.5498	0.0237	2.5785
WEAVING MANEUVER	Without EKF	7.0045	3.3466	0.2376	11.0837
on single plane	With EKF and decreasing R	0.1385	0.1012	0.0152	0.6447
$n_{T_y} = 5g$, $n_{T_z} = 0g$	With EKF and decreasing R	0.8568	0.2972	0.1661	1.3249
WEAVING MANEUVER	Without EKF	8.6237	4.1013	0.4896	13.0811
on both planes	With EKF and decreasing R	0.1368	0.1321	0.0144	0.7881
$n_{T_y} = 5g$, $n_{T_z} = -4g$	With EKF and decreasing R	1.0676	0.4008	0.1107	1.8471
WEAVING MANEUVER	Without EKF	3.7598	1.2463	0.6835	7.1193
on single plane	With EKF and decreasing R	0.3292	0.2257	0.0232	0.9368
$n_{T_y} = -4g$, $n_{T_z} = 0g$	With EKF and decreasing R	0.4720	0.3880	0.0228	1.5762
WEAVING MANEUVER	Without EKF	3.5552	1.7322	0.2150	7.9577
on both planes	With EKF and decreasing R	0.3312	0.2811	0.0152	1.2122
$n_{T_y} = -4g$, $n_{T_z} = -2g$	With EKF and decreasing R	0.6007	0.5433	0.0350	3.2305

Figure 3 shows the Miss distance Cumulative Distribution Function for a step maneuver on both planes. In 90% of the cases the miss distance values do not exceed $6.5m$ without using the EKF filter, $1.2m$ using the EKF filter with a constant matrix R and $0.50m$ using the EKF filter with a decreasing matrix R.

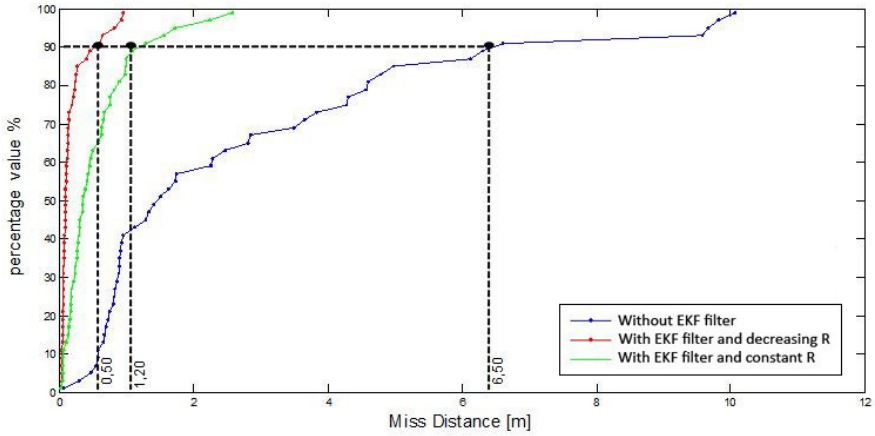

Fig. 3. Miss distance Cumulative Distribution for a STEP MANEUVER on both planes.

6. Conclusions

The use of a Proportional Navigation law for a missile can introduce significant miss distance in the presence of a maneuvering target. For improving the terminal homing an Extended Kalman Filter is developed to estimate the guidance signals. The implemented EKF introduces good estimations of relative positions, relative velocities and target accelerations in a 3D engagement between a pursuer and an evader. These estimations provided optimal guidance signals that can be used with the Proportional Navigation law.

In all considered scenarios the EKF led to a remarkable improvement: the miss distance values are much smaller than the cases in which the EKF was not applied. In addition the use of a matrix R linearly decreasing with the range led to further improvement with respect to the use of a constant matrix R. Moreover, the use of the EKF filter in conjunction with the Advanced Guidance law yields performance superior to that of Proportional Navigation guidance system.

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