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Optimal Trajectories For Spacecrafts with M2P2 Propulsion System

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Abstract

Mini Magnetospheric Plasma Propulsion (M2P2) is a propulsion system based on the deflection of the ambient plasma through the deployment of a magnetic bubble surrounding the spacecraft. Although the plasma momentum is small, it is possible to obtain even some newtons of thrust through the employment of a large magnetic field. After a description of the dynamic model of thrust, a discussion of the performances of M2P2 in comparison with other propulsion systems is given. Results of comparisons are expressed through some test cases.

Keywords: `satellite; optimization; interplanetary trajectories`

1. Introduction.

Mini Magnetospheric Plasma Propulsion is a kind of propulsion system based upon the deflection of the ambient plasma through the employment of an intense magnetic field surrounding the spacecraft.

At a given distance from the spacecraft, depending upon the magnetic field strength and the solar wind pressure, the plasma is diverted from its original trajectory and some momentum is transferred to the spacecraft. The radius of this interaction region, which finally depicts a sort of bubble surrounding the spacecraft, can be simply estimated by comparing the magnetic pressure and the solar wind pressure: the distance at which the first one equals the second one gives an approximate value of the radius of the interaction region. The momentum is transferred to the spacecraft through this magnetic bubble; it is evident that the large is the interaction radius, the bigger is the thrust level attainable.

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A dipolar magnetic field by itself would not be able to produce an interaction region enough wide to produce a relevant thrust level; the basic concept behind the M2P2 propulsion system is to limit the magnetic field decrease along with the distance through the injection of plasma into the field lines. The effect of plasma is to stretch the dipole lines so that the magnetic field intensity decrease as $R^{-1.1}$ instead of R^{-3}, being R the distance from the dipole. Plasma consuption is in the order of 0.5 kg/day.

Among the disadvantages of the M2P2 system the most relevant is the impossibility to modify the thrust direction, which is aligned with the solar wind velocity vector. This features makes the M2P2 unsuitable, for example, to perform interplanetary transfers, unless it is supported by another kind of propulsion system such as a rocket engine. Also the high level of power required onboard to produce the magnetic field is another negative aspect related to the employment of M2P2 system in comparison with others.

2. The thrust model.

Thrust modulus and direction of M2P2 depend upon the interaction with the inflated magnetosphere with the solar wind flux. By looking at the Parker model for the solar wind, it is possible to assume that solar wind is ejected by Sun corona in all the space directions with supersonic speed and that the interaction with the magnetosphere produces a thrust which is mostly oriented along the Sun-spacecraft direction.

In this work two different thrust models have been considered: a purely radial thrust model, in which the only variable parameter is the thrust magnitude (which can be null or equal to a finite value), and a model where the acceleration direction can slightly changes within a cone whose axis is aligned with the Sun-spacecraft conjunction line. In this second model the thrust magnitude depends upon the tangential component of the acceleration, and the thrust components depend upon each other according a polynomial expression. Given an heliocentric reference frame and assuming the spacecraft depicts a trajectory laying in the ecliptical plane, let $\hat{r}$, $\hat{\vartheta}$ the two unit vectors describing the radial (Sun-spacecraft line) and the tangential direction respectively. In such a reference frame, the acceleration components provided by M2P2 would be represented by the following relations:

$$f_r = \tau \frac{T}{m}, f_\vartheta = 0 \tag{1}$$

being T the thrust maximum value, m the spacecraft mass and τ an on-off

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function. As for the second model, one gets:

$$f_r = f_{r,0}\left[1 + a_1\left(\frac{f_\vartheta}{f_{r,0}}\right) + a_2\left(\frac{f_\vartheta}{f_{r,0}}\right)^2\right], f_\vartheta \neq 0 \tag{2}$$

where the coefficients a_1, a_2 are specific for the given magnetosphere.

In the following section a detailed description of the test cases and of the corresponding equations of motion will be given.

3. Analysis of the M2P2 performances.

3.1. *Test Case 1: minimum time interplanetary transfer*

As first test case a typical problem of interplanetary transfer has been chosen: the minimum time Earth-Mars transfer. This problem is particularly interesting, as Mars represents the last frontier to the manned space exploration. The most critical problem concerning an interplanetary transfer to Mars is the time needed to reach the target with a given propulsion system: the prolonged exposure to radiation and the absence of gravity makes the travel deleterious for human health. For this reason the minimum time problem has been chosen to analyze the M2P2 performances. Also a minimum time transfer to Jupiter might be of interest for scientific missions: in this work, both cases have been considered. To note that M2P2 propulsion system is used only to perform the interplanetary transfer, and not to provide the thrust required for the escape from Earth and the capture into the target body gravity field. In both cases the employment of M2P2 would be by far unprofitable, since the acceleration due to the magnetic bubble is too small in comparison with the gravity one acting on the spacecraft when it is inside the planet sphere of influence. Escaping the planet gravity or spiralizing towards the destination orbit would require an amount of time whose order of magnitude is similar to that of the time nessary to perform the interplanetary transfer. Therefore, the injection into the escape hyperbola and the capture manoeuvre must be executed by means of a chemical propulsion system. More in detail, the spacecraft could be put straight into the escape hyperbola by the launcher upper stage. This would lead to a further reduction of the spacecraft mass.

In this work the problem of the minimum transfer time is solved through the well-known approach given by the Lagrange multipliers, leading to the resolution of the so-called Mayer problem according to the Potryagin maximum principle. The Mayer problem is a particular case of the more general Bolza problem, which can be summarized as follows. Given a n-dimensional, non autonomous system of first order differential equations controlled by the r-dimensional action $\mathbf{u}$:

$$\mathbf{F}(\mathbf{z}, \dot{\mathbf{z}}, \mathbf{u}, t) = 0 \tag{3}$$

where $\mathbf{F} : \mathbb{R}^n \times \mathbb{R}^n \times RR^p \times RR \longrightarrow RR^n$, a functional:

$$J = \int_{t_1}^{t_2} g(\mathbf{z}, \dot{\mathbf{z}}, \mathbf{u}, t) dt + h\left[t_1, \mathbf{z}(t_1), t_2, \mathbf{z}(t_2)\right] \tag{4}$$

with $g : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R} \longrightarrow \mathbb{R}$, $g : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R} \longrightarrow \mathbb{R}$, and a set of boundary conditions:

$$\psi\left[t_1, \mathbf{z}(t_1), t_2, \mathbf{z}(t_2)\right] = 0 \tag{5}$$

being $\psi : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n$, $p \leq 2n + 2$ the Bolza problem consists of the minimization of the functional J under the differential constraints given by the system $\mathbf{F}(\mathbf{z}, \dot{\mathbf{z}}, \mathbf{u}, t) = 0$, so that also boundary conditions $\psi\left[t_1, \mathbf{z}(t_1), t_2, \mathbf{z}(t_2)\right] = 0$ are respected. In other terms, the solution to the Bolza problem is the trajectory, among the all possible ones connecting $\mathbf{z}(t_1)$ to $\mathbf{z}(t_2)$, which makes minimum the functional J. If one states that:

$$J = h\left[t_1, \mathbf{z}(t_1), t_2, \mathbf{z}(t_2)\right] \tag{6}$$

and

$$p < 2n + 2 \tag{7}$$

the Bolza problem is known as the Mayer problem. Condition 6 implies that the functional depends upon the solely initial and final required conditions, whereas condition 7 states that boundary constraints interest only a part of the state vector, and not the whole one.

In the calculus of variations, necessary conditions for optimality can be obtained through Lagrange multipliers; the relations describing such necessary conditions for a strong maximum are given by the Pontryagin maximum principle, recalled below. Let the system be autonomous, so that:

$$\mathbf{F}(\mathbf{z}, \dot{\mathbf{z}}, \mathbf{u}) = 0 \tag{8}$$

or, in a more useful way:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mathbf{u}) \tag{9}$$

with $f : \mathbb{R}^n \times \mathbb{R}^p \longrightarrow \mathbb{R}^n$. Named with Λ the vector of Lagrange multipliers, let

$$H\left[\Lambda, \mathbf{f}(\mathbf{z}, \mathbf{u})\right] \tag{10}$$

be the Hamiltonian of the controlled system. The Pontryagin maximum principle states that if $\mathbf{u}^*, \mathbf{z}^*$ are solution of the optimal control Mayer problem, then:

$$\exists \Lambda \in \mathbf{C}^1, \Lambda \neq 0 : \forall t \in [t_1, t_2], \dot{\mathbf{z}} = \frac{\partial H}{\partial \Lambda}, \dot{\Lambda} = -\frac{\partial H}{\partial \mathbf{z}} \tag{11}$$

and

$$\forall t \in [t_1, t_2]\, H[\Lambda, \mathbf{f}(\mathbf{z}^*, \mathbf{u}^*)] = sup\{H[\Lambda, \mathbf{f}(\mathbf{z}^*, \mathbf{u}^*)] : \mathbf{u} \in \mathbf{U}\} = M[\Lambda, \mathbf{z}^*(t)] \tag{12}$$

Besides, the transversality conditions must be added. Such principle gives only necessary conditions for optimality. After the concise introduction explaining some basic concepts about the optimal control theory, it is possible to write the system of motion equations for the analysis of the transfer trajectories. Transfers has been analyzed under the following hypothesis: 1) Earth and Mars (Jupiter) orbits are circular and coplanar; 2) The transfer segment begins at 1.00 A.U. (the mean Earth orbit radius); 3) The transfer segment ends once the spacecraft reaches 1.52 A.U. (the mean Mars orbit radius) or 5.24 A.U. (the mean Jupiter radius), being the spacecraft velocity parallel to the target one.

Even if such hypothesis could appear too strict for the general problem of transfer optimization, the results attainable allow to carry out satisfying estimations of rockets performances.

The first-order differential equation system for the co-planar transfer problem is thus the following:

$$\begin{cases} \frac{dr}{dt} = \dot{r} \\ \frac{d\vartheta}{dt} = \dot{\vartheta} \\ \frac{d\dot{r}}{dt} = \dot{\vartheta} r^2 - \frac{\mu}{r^2} + \frac{f_r}{m} \\ \frac{d\dot{\vartheta}}{dt} = -2\dot{\vartheta}\frac{\dot{r}}{r} + \frac{f_\vartheta}{m} \\ \frac{dm}{dt} = -\tau\,|k| \end{cases} \tag{13}$$

being r, ϑ the spacecraft polar coordinates in an heliocentric reference frame, $\dot{r}, \dot{\vartheta}$ their temporal derivatives, m the mass value, μ the Sun gravitational parameter, f_r, f_ϑ the thrust components, τ an on-off switch function and finally k represents the absolute mass variation with respect to time. Analogies with the formalism introduced in the previous paragraph are evident: the state vector $\mathbf{z}$ consists of $\left\{r, \vartheta, \dot{r}, \dot{\vartheta}, m\right\}$, $\left\{\frac{dr}{dt}, \frac{d\vartheta}{dt}, \frac{d\dot{r}}{dt}, \frac{d\dot{\vartheta}}{dt}, \frac{dm}{dt}\right\}$ are the components of the $\dot{\mathbf{z}}$ vector and finally $\{f_r, f_\vartheta\}$ represent the force vector $\mathbf{u}$. It is important to notice that $\{f_r, f_\vartheta\}$ values can be expressed by:

$$f_r = \tau\frac{T}{m}, f_\vartheta = 0 \tag{14}$$

assuming a purely radial thrust, or by:

$$f_r = f_{r,0}\left[1 + a_1(\frac{f_\vartheta}{f_{r,0}}) + a_2(\frac{f_\vartheta}{f_{r,0}})^2\right], f_\vartheta \neq 0 \tag{15}$$

if a small dipole tilt is allowed.

Beside the differential equation system 13, it is necessary to include also the derivatives of the Lagrange multipliers so that conditions stated by the Pontryagin maximum principle are satisfied. Let $\{\lambda_r, \lambda_\vartheta, \lambda_{\dot{r}}, \lambda_{\dot{\vartheta}}, \lambda_m\}$ be the components of the Λ vector and so the Lagrange multipliers of $\frac{dr}{dt}, \frac{d\vartheta}{dt}, \frac{d\dot{r}}{dt}, \frac{d\dot{\vartheta}}{dt}, \frac{dm}{dt}$ respectively; let also be

$$J = \int_{t_0}^{t_f} L, dt = t_f - t_0 \tag{16}$$

the cost function to be optimized, with $L = 1$, according to the particular problem chosen for the analysis. The Hamiltonian function H shall therefore assume the form:

$$H\left[\Lambda, \dot{\mathbf{z}}\right] = \lambda_r \frac{dr}{dt} + \lambda_\vartheta \frac{d\vartheta}{dt} + \lambda_{\dot{r}} \frac{d\dot{r}}{dt} + \lambda_{\dot{\vartheta}} \frac{d\dot{\vartheta}}{dt} + \lambda_m \frac{dm}{dt} + L \tag{17}$$

Once obtained an expression for H and J, it is necessary to prove that conditions 11 and 12 posed by the Pontryagin principle have been satisfied. First of all, let us consider the expression 11. It is evident that such formulation satisfies condition $\dot{\mathbf{z}} = \frac{\partial H}{\partial \Lambda}$, as can be easily showed. To meet the further conditions according to $\dot{\Lambda} = -\frac{\partial H}{\partial \mathbf{z}}$ it shall be:

$$\begin{cases} \frac{d\lambda_r}{dr} = -\frac{\partial H}{\partial r} = -\lambda_{\dot{r}}\left(\dot{\vartheta}^2 + 2\frac{\mu}{r^3}\right) - \lambda_{\dot{\vartheta}}\left(2\dot{\vartheta}\frac{\dot{r}}{r^2} - \frac{f_\vartheta}{r^2 m}\right) \\ \frac{d\lambda_\vartheta}{dt} = -\frac{\partial H}{\partial \vartheta} = 0 \\ \frac{d\lambda_{\dot{r}}}{dt} = -\frac{\partial H}{\partial \dot{r}} = -\lambda_r \\ \frac{d\lambda_{\dot{\vartheta}}}{dt} = -\frac{\partial H}{\partial \dot{\vartheta}} = -\lambda_\vartheta \\ \frac{d\lambda_m}{dt} = -\frac{\partial H}{\partial m} = \frac{1}{m^2}\left(\lambda_r f_r + \lambda_\vartheta f_\vartheta\right) \end{cases} \tag{18}$$

Until now only condition 11 has been verified. To satisfy the constraint given by 12, the thrust components f_r, f_ϑ must be proportional to the $\lambda_{\dot{r}}, \lambda_{\dot{\vartheta}}$ Lagrange multipliers: of course such condition, which holds when thrust direction can be varied, cannot be applied as is when considering M2P2. By assuming a purely radial thrust, the condition 12 turns into:

$$\begin{cases} \tau = 1, if\left(\lambda_r T - \lambda_m \left|k\right|\right) > 0 \\ \tau = 1, if\left(\lambda_r T - \lambda_m \left|k\right|\right) \leq 0 \end{cases} \tag{19}$$

If the second model is considered, the thrust component f_r, f_ϑ must be proportional to the $\lambda_{\dot{r}}, \lambda_{\dot{\vartheta}}$ only if:

$$\arctan\left(\left|\frac{f_\vartheta}{f_r}\right|\right) < \delta \tag{20}$$

being δ the cone angle within which thrust direction is confined. Let us assume $0 \leq f_\vartheta \leq f_{\vartheta,max}$; let also f_ϑ^* be the optimal value of f_ϑ. If one defines:

$$\begin{cases} A = \tau \frac{\lambda_{\dot{r}}}{m} \frac{a_2}{f_{r,0}} \\ B = \tau \left(\frac{\lambda_{\dot{r}}}{m} a_1 + \frac{\lambda_{\dot{\vartheta}}}{rm}\right) \\ C = \tau \left(\frac{\lambda_{\dot{r}}}{m} f_{r,0} - \lambda \left|k\right|\right) \\ H^* \left(f_\vartheta\right) = A \left(f_\vartheta^*\right)^2 + B \left(f_\vartheta^*\right) + C \end{cases} \tag{21}$$

optimal f_ϑ can be evaluated easily as follows:

$$\tau \neq 0 \Rightarrow f_\vartheta^* = \begin{cases} -\frac{B}{2A}, if A < 0\, H^* \left(-\frac{B}{2A}\right) > 0 \\ 0, if A > 0\, H^* \left(0\right) > max\left[H\left(f_{\vartheta,max}\right), 0\right] \\ f_{\vartheta,max}, if A > 0\, H^* \left(f_{\vartheta,max}\right) > max\left[H\left(0\right), 0\right] \end{cases} \tag{22}$$

As for the transversality conditions, it shall be:

$$H\left(t = t_0\right) = 0 \tag{23}$$

A proper choice of the initial values of the Lagrange multipliers shall ensure that 11, 12 and the transversality conditions are satisfied.

3.2. *Test Case 2: escape from the Solar System*

The radial, continuous thrust allows to reach quickly the escape conditions from the Solar System; it is also possible to increase the hyperbolic excess by keeping an high magnetic field around the spacecraft, so to receive a further acceleration through the M2P2 system even after the escape conditions have been reached. This allows to reduce the time needed to reach the Solar System boundaries, as it will be shown in the following Section.

4. Results and discussion

4.1. *Test Case 1: minimum time interplanetary transfer*

As for the interplanetary transfer towards Mars, Fig. 4.1 shows some transfer trajectories by changing the value of the initial spacecraft mass, in the range between 100 - 1000 kg. The "firing" period ranges between 6

- 65% of the overall transfer time, according to the initial mass value. Fig. 4.1 shows the time needed to perform the trip. It is evident that by rising the spacecraft mass, also the transfer time increases; in the considered mass range, the transfer time is always lower than that required for an Homhann transfer, equal to 259 days. To be noticed that a further ΔV is required to permit the spacecraft injection into a capture trajectory, since the M2P2 acceleration is almost completely along the radial direction. Assuming an initial spacecraft mass equal to 1000 kg, such impulse along the spacecraft velocity vector would be equal to 4.52 km/sec (to note that the overall ΔV for an Homhann transfer is equal to 5.59 km/sec). This implication makes M2P2 propulsion system not suitable for interplanetary transfers. Analogous considerations can be done as for the transfer to Jupiter.

4.2. *Test case 2: escape from the Solar System*

In Fig. 4.2 a plot of the escape trajectories is given; results from simulations are shown in Fig. 4.2. As for the escape from the Solar System, M2P2 system allows to maintain a radial constant acceleration until plasma has been consumed completely. For these simulation plasma mass percentage was assumed to be 30% of the whole spacecraft mass; this hypothesis is in full agreement with the system requirements and it has been already verified in other papers. For a 1000 kg spacecraft, the time needed to reach Pluto orbit is about 6.6 years, with a plasma consumption equal to the 30% of the initial spacecraft mass. An impulsive manoeuvre for the injection into an escape parabola would require a ΔV of 12 km/sec, corresponding to 930 kg of propellant.

5. Conclusions

The M2P2 is a propulsion system allowing to execute manoeuvre requiring a thrust level in the range (0.1 - 1 N) with a reduced propellant (plasma) consumption. As the simulations showed, such propulsion system is not suitable for interplanetary transfer, since the acceleration it is able to provide is mostly in the radial direction. As for the escape from the Solar System, M2P2 proved to be efficient, since no tangential manoeuvre is required. Mass consumption and the required time are sensibly smaller than those necessary when employing a chemical propulsion system.

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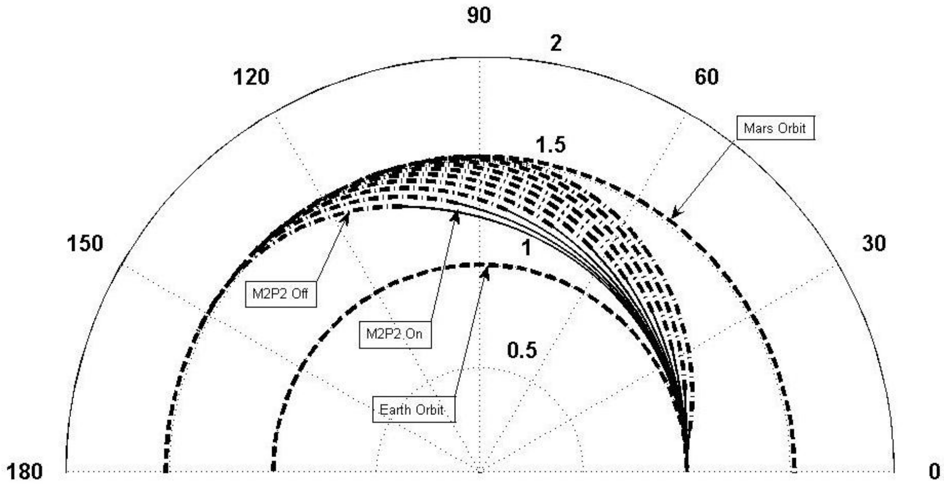

Fig. 1. Transfer trajectories towards Mars according to different values of the starting spacecraft mass, in the range 100 - 1000 kg. The dashed segments in the figure represent the trajectory arches where M2P2 was active. Thrust is assumed to be along the radial direction.

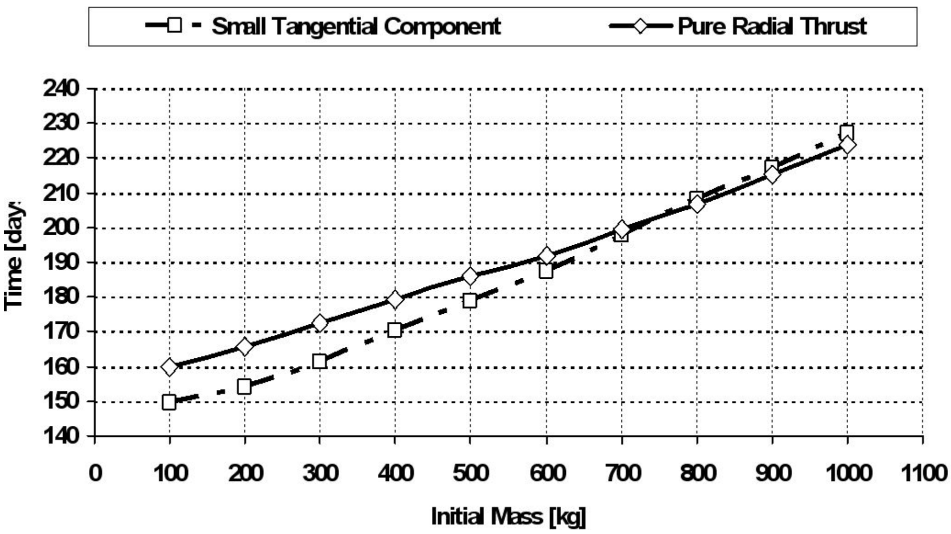

Fig. 2. Comparison between the transfer time vs the initial spacecraft mass, considering the two different thrust models. No relevant differences have been observed.

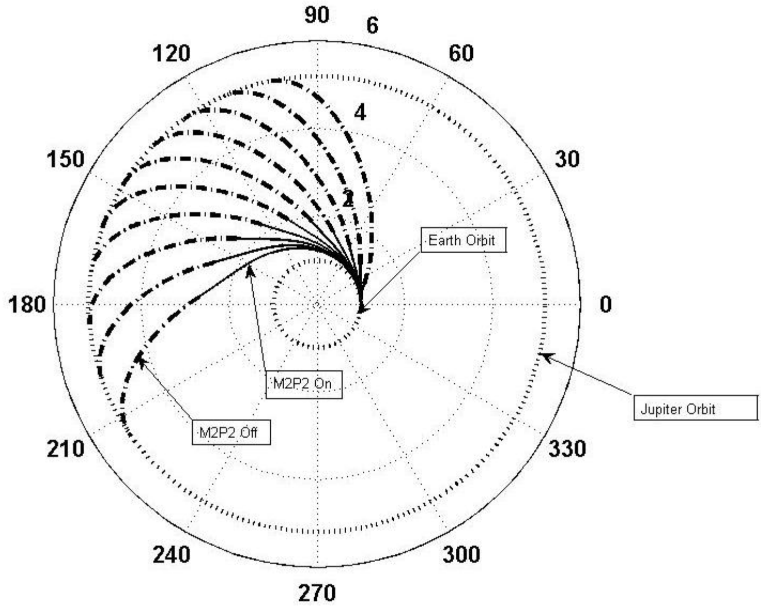

Fig. 3. Transfer trajectories towards Jupiter according to different values of the starting spacecraft mass, in the range 100 - 1000 kg. The dashed segments in the figure represent the trajectory arches where the M2P2 was active. Thrust is assumed to be along the radial direction.

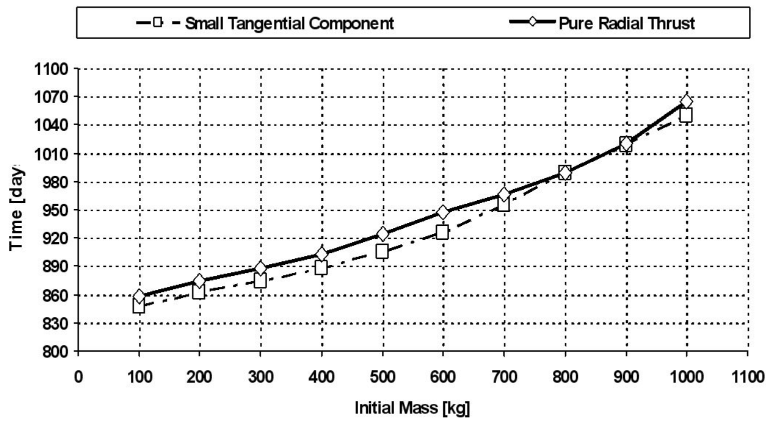

Fig. 4. Comparison between the transfer time vs the initial spacecraft mass, in the range 100 - 1000 kg, considering the two different thrust models. No relevant differences have been observed.

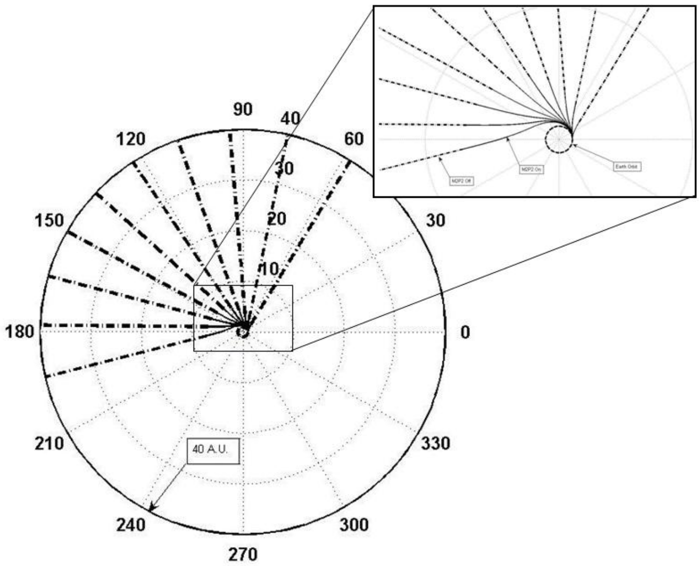

Fig. 5. Escape trajectories according to different values of the starting spacecraft mass, in the range 100 - 1000 kg. The dashed segments in the figure represent the trajectory arches where the M2P2 was active. In the enlargement, the early phase of the transfer.

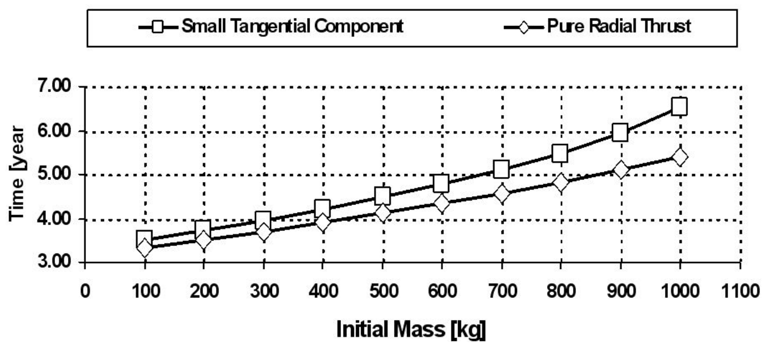

Fig. 6. Comparison between the escape time vs the initial spacecraft mass, considering the two different thrust models. No relevant differences have been observed.