Communications to SIMAI Congress, DOI: 10.1685/CSC09270
ISSN 1827-9015, Vol. 3 (2009) 270 (12pp)

An analytic tool for performance evaluation of air-dropped launchers

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Abstract

The optimization of trajectories performed by air-dropped launcher is a new subject in spaceflight optimization problems. Some analytical formulas are here used to drive numerical algorithms to convergence to optimal solutions. The joint use of analytical formulas (to provide a starting guess) and numerical iterative algorithms provides a flexible tool able to find optimal trajectories for different configurations of launchers dropped with different flight conditions.

Keywords: `Optimization, airdropped launchers, analytic formulas`

1. Introduction.

The optimization of space trajectories consists in the definition of a thrust strategy (control variables: thrust magnitude and thrust direction) to achieve the final orbit from a given initial state, while optimizing some cost function, for instance fuel consumption. The optimal thrust strategy is generally found by numerical and iterative algorithms that require a starting guess for the solution and are local in character. Namely these problems are generally solved introducing Lagrange multipliers and the calculus of variation (or Pontryagin Principle) to determine necessary conditions that must be satisfied by an optimal solution [1]. The determination of the control variables from the necessary conditions results in a Two–Point Boundary–Value Problem (TPBVP), since some of the state variables are specified at the initial time t_0, whereas other ones are given at the final time t_f (required final conditions).

The TPBVP usually can not be solved analytically so numerical methods are to be employed. Shooting methods represent a class of numerical methods for solving such problems. Starting guess are given to the unknown variables, generally the initial values of the Lagrangian multipliers, then the

Received 13/02/2009, in final form 03/07/2009
Published 31/07/2009

required final conditions are checked. If they are not satisfied to the prefixed accuracy, the initial guess is improved after evaluating the dependence of the boundary condition violation on the starting guess. A well known drawback characterizes shooting methods, i.e. their poor robustness. As a matter of fact, the basin of attraction of an optimal solution can be very narrow, so a suitable guess must be provided to achieve the convergence to the final (optimal) result. In addition, Lagrangian multipliers usually do not have a straightforward physical meaning and their dependence on the initial values of the state variables can be strongly nonlinear.

The determination of suitable initial conditions is therefore a critical point in the search of an optimal solution and the success of the procedure depends very much on the skill and experience of the user. Generally a sort of homotopic procedure is followed, that is one starts from the solutions (unknown initial conditions) of ”near-by-problems” to move continuously to the problem in hands. A difficulty emerges when a completely new concept, in terms of launcher configuration or mission to be achieved, must be treated. In these case new approaches to provide a reasonable guess of the optimal solution must be employed.

Recently the use of air launched rockets released from high performance aircraft have been proposed to deliver microsatellites in low Earth orbits. These launch systems need a reduced amount of operations and ground support, thus offering launch on demand capability, with the possibility to set payloads into orbit within very short time. In addition such systems are more reliable under unfavorable weather conditions. Fast response and reliability are crucial issues when space support is required in case of humanitarian emergencies, conflicts, natural disasters, etc.

This concept was firstly introduced in [2] using the F-404 as carrier aicraft, then the European Fighter Aircraft (EFA) was proposed in [3] to set into low orbits payloads up to 50 Kg. The same payload weight is achieved using the Rafale fighter aircraft, however the payload weight can be doubled using an advanced concept [4]. The F-15 was proposed in [5] as carrier aircraft for a launcher released from the upper part of the fuselage. The MiG 25 was proposed in [6] and the Douglas F-4D in [7].

A commune problem of these projects is the determination of the maximum mass that can be set into low orbits while satisfying the severe constraints imposed on rockets air-launched from a high performance aircraft. In particular:

1. The total weight that can be released from the aircraft is fixed to a maximum value, taking into account also the need of additional tanks to increase the aircraft range

2. The length of the rocket is limited to the space between the aircraft engine and the landing gear
3. The rocket diameter is constrained to a maximum value, due to the height of the aircraft and the clearance space for the take-off maneuver.

These constraints represent the central problem while trying to design the launcher using stages of existing rockets, so the trajectory optimization must be carried out together with the design of a new launcher. Moreover, variable release conditions can be considered in the optimization process, and these conditions are directly related to the total mass and to the configuration (winged or non-winged) of the rocket [8].

Since the optimization of such a launch system is a complex and new problem, a fast tool for a preliminary design of the launch mission is rather helpful. Such a tool is presented here for the first time and it has been already used for the design of optimal launch trajectories of rockets released from different high performance aircraft. The tool is based on a symple numerical algorithm relaying on some analytic formulas.

In Section 2 the analytical formulas used for the numerical calculus are introduced. In Section 3 the numerical algorithm is presented, and in Section 4 the analysis is applied to rockets released from the European Fighter Aircraft. Final remarks conclude the paper.

2. Preliminary Rocket and Trajectory Design

The equation of motion of a rocket are simplified to get a first guess solution that can be used in a numerical iterative algorithm of optimization. For the definition of the engine performance of a rocket the following basic parameters are considered:

1. Initial thrust to weight ratio $n_0 = \frac{T}{gm_0}$, where T is the thrust (regarded as constant) and m_0 is the initial mass
2. Specific impulse Isp (in seconds)
3. Structural mass ratio $u = \frac{m_s}{m_0}$, where m_s is the mass of the structure
4. Surface to mass ratio $\beta = \frac{1}{2}\frac{S}{m_0}$ with $S = \frac{\pi}{4}d^2$, d being the rocket diameter. These four fundamental parameters must be specified for each stage of the rocket.

In the hypothesis of a planar launch trajectory and gravity turn guidance (that is the thrust is always directed along the velocity direction), the equation of motion are:

$$(1) \qquad \begin{cases} \dot{V} = g\dfrac{n_0}{1-qt} - \dfrac{\mu}{r^3}\sin\gamma - \rho\dfrac{\beta}{1-qt}C_D V^2 \\ \dot{\gamma} = -\dfrac{1}{V}\dfrac{\mu}{r^2}\cos\gamma + \dfrac{V}{r}\cos\gamma \\ \dot{r} = V\sin\gamma \\ \dot{\theta} = \dfrac{V}{r}\cos\gamma \end{cases}$$

where V , γ are the absolute velocity and flight path angle, r, θ are the polar coordinates of the rocket center of mass. The specific mass rate $q = \frac{n_0}{Isp}$ is defined and the trust term must be considered till the engine burn out time $t_b = \frac{1-u}{q}$. The drag coefficient C_D is function of the Mach number and the exponential model of the density is considered : $\rho = \rho_0 e^{-\beta_h h}$ (h is the altitude). To make the problem amenable to an analytic solution the following assumptions are given :

a) Constant gravitational field
b) No atmosphere
c) Constant thrust to weight ratio, so the average value $\overline{n} = \frac{n_0}{1-u} log\frac{1}{u}$ will be considered.

In these hypothesis the following formulas holds true [9]:

$$(2) \qquad V = Az^{\overline{n}-1}\left(1+z^2\right)$$

$$(3) \qquad t - t_0 = \frac{A}{g}z^{\overline{n}-1}\left(\frac{1}{\overline{n}-1} + \frac{z^2}{\overline{n}+1}\right) - \frac{A}{g}z_0^{\overline{n}-1}\left(\frac{1}{\overline{n}-1} + \frac{z_0^2}{\overline{n}+1}\right)$$

$$(4) \qquad h - h_0 = \frac{A^2}{g}\left[\frac{z^{2\overline{n}-2}}{2\overline{n}-2} - \frac{z^{2\overline{n}+2}}{2\overline{n}+2}\right]_{z_0}^{z}$$

where the variable z is related to the flight path angle γ:

$$z = tan\frac{\chi}{2}\ , \quad \chi = \frac{\pi}{2} - \gamma$$

The initial flight conditions (the dropping conditions) V_0, γ_0 determine the constant A of formula (2). Moreover the value of the flight path angle at the burn out time t_b, γ_f can be obtained by formula (4) that must be solved with respect to z_f:

$$t_b = \frac{A}{g} z_f^{\overline{n}-1}\left(\frac{1}{\overline{n}-1}+\frac{z_f^2}{\overline{n}+1}\right) - \frac{A}{g} z_0^{\overline{n}-1}\left(\frac{1}{\overline{n}-1}+\frac{z_0^2}{\overline{n}+1}\right) \tag{5}$$

These formulas can be generalized to multistage rockets. For instance, if a two stage rocket is considered, the velocity of the second stage is :

$$V_2 = A_2 z_2^{\overline{n}_2-1}\left(1+z_2^2\right) \tag{6}$$

where the constant A_2 and the initial condition for the z_2-variable must satisfy the equations (continuity conditions):

$$\frac{A_1}{A_2} = z_{20}^{\overline{n}_2-\overline{n}_1} \tag{7}$$

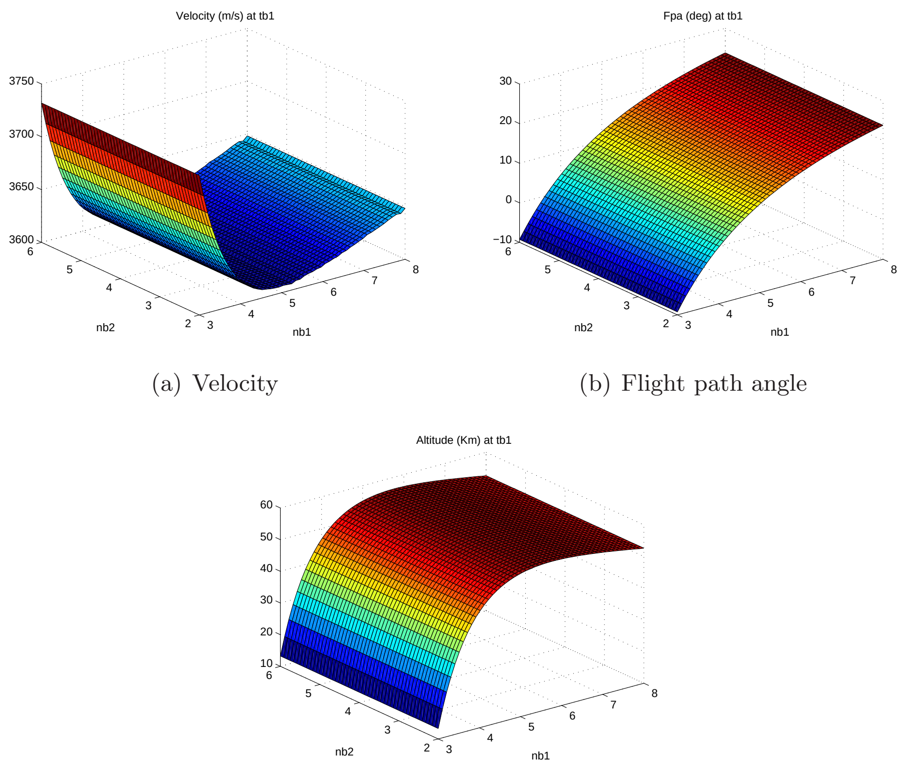
$$z_{20} = z_{1f} \tag{8}$$

The equation (4) is now applied at second stage burn out $t_{b_2} = \frac{1-u_2}{q_2}$:

$$t_{b_2} = \frac{A_2}{g} z_{2f}^{\overline{n}_2-1}\left(\frac{1}{\overline{n}_2-1}+\frac{z_{2f}^2}{\overline{n}_2+1}\right) - \frac{A_2}{g} z_{1f}^{\overline{n}_2-1}\left(\frac{1}{\overline{n}_2-1}+\frac{z_{1f}^2}{\overline{n}_2+1}\right) \tag{9}$$

and it can be solved with respect to z_{2f}, so the flight path angle γ_{2f} at the end of the second stage can be determined.

The general rocket design can be approached using the above formulas. The best final conditions, for instance maximum velocity, can be determined as function of the fundamental parameters of each rocket stage $n_{0k}, Isp_k, u_k, \beta_k$ and release state conditions V_0, γ_0, h_0. Of course some constraints must be taken into account: the total mass m_0 of the rocket and the release conditions are constrained by the carrier aircraft capability. The diameter of the rocket is also fixed, so the value β_1 is fixed. The structural mass ratio u_k can be fixed to the values 0.12 for solid propellant stages and 0.33 for liquid propellant stages. The specific impulses can be also fixed to the characteristic values $Isp = 280\,s$ for solid and $Isp = 300\,s$ for liquid stages. Then, the dependence of V_f, γ_f, h_f on the (average) thrust to weight ratios n_{b_1}, n_{b_2} and initial conditions V_0, γ_0 can be inspected looking the plots obtained by the above formulas. For instance Figures 1 and 2 show the values of velocity, flight path angle and height achieved at the first and second stage burn out times t_{b_1}, t_{b_2} for different average thrust to weight



(a) Velocity

(b) Flight path angle

(c) Altitude

Fig. 1. Velocity $[m/s]$ (a), Flight path angle $[deg]$ (b) and Altitude $[Km]$ (c) at t_{b_1} for different average thrust to weight ratios

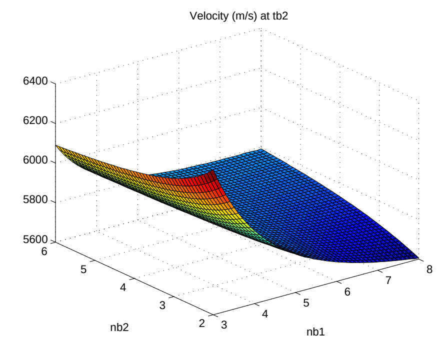
ratios related to the first and second stage ($\overline{n}_1, \overline{n}_2$ respectively). The drop conditions are fixed to $V_0 = 280m/s$ and $\gamma_0 = 40\ deg$.

Some remarks are in order:

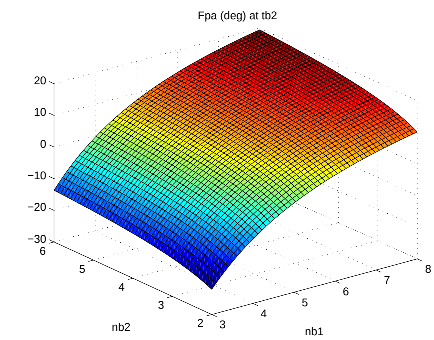
Remark 2.1. Of course the data at t_{b_1} do not depend on the second stage thrust to weight value $\overline{n}_2$. Higher values of velocity correspond to lower values of altitude, since the gravitational loss and the altitude rate both depend on $\sin\gamma$. A compromise between the maximum altitude and maximum velocity has to be found.

Remark 2.2. The Figure 1(c) show that the altitude achieved at the first stage burn out time stays below 55 Km for any value of $\overline{n}_{b_1}$, $\overline{n}_{b_2}$. This suggest that no coasting time is useful between the first and the second stage

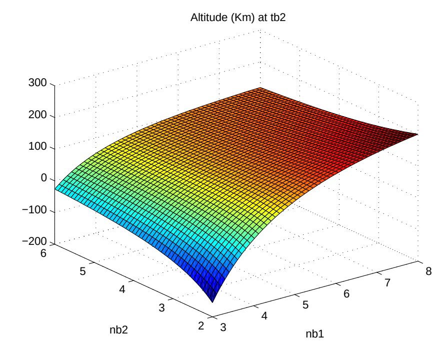
DOI: 10.1685/CSC09270



(a) Velocity



(b) Flight path angle



(c) Altitude

Fig. 2. Velocity $[m/s]$ (a), Flight path angle $[deg]$ (b) and Altitude $[Km]$ (c) at t_{b_2} for different average thrust to weight ratios

because too high drag losses would occur.

Remark 2.3. The dependence of both the velocity and altitude on the average thrust to weight ratio of the second stage is rather weak, in particular for lower values of $\overline{n}_{b_1}$. This give indications on the thrust and mass distribution between the two stages. In particular it is not worth to consider a second stage with large $\overline{n}_{b_2}$.

Remark 2.4. The flight path angle at t_{b_1} must be definitely bigger than zero. This implies that $\overline{n}_{b_1}$ must be definitely bigger than 4

The inspection of these Figures, as well as similar Figures obtained with different dropping conditions V_0, γ_0, allows to determine suitable values of $\overline{n}$, hence of n_0, for each of the stages. These choices determine also the values of velocity, flight path angle and altitude, in particular at the burn out times.

For instance, let the average thrust to weight ratios be fixed to the values $\overline{n}_1 = 6$, $\overline{n}_2 = 2.5$. Figure 3 shows the dependence of the state variables at the burn out times t_{b_1}, t_{b_2} for different initial drop conditions γ_0, V_0 being fixed to 300 m/s. It turns out that the rocket must be released with initial flight path angle bigger than 35 deg. The above data, found by the analytic formulas give suitable input to the numerical algorithms.

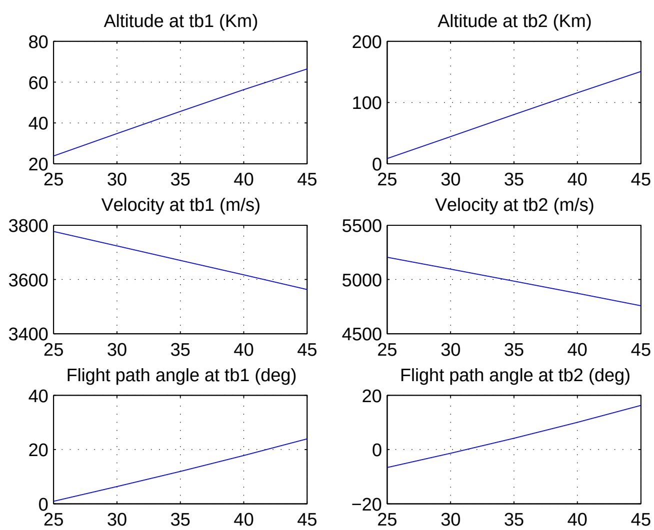


Fig. 3. State variables at burn out times for different γ_0

3. Trajectory Optimization

The optimal trajectories are designed using the preliminary rocket and trajectory design as input for a numerical optimization code. The input data are the rocket geometry, mass and thrust parameters as well as the slope of the trajectory at the burn out of any stage. That is the four fundamental parameters for each stage n_k, Isp_k, u_k, β_k are determined from the above preliminary analysis, as well as the flight path angles γ_{kf} defined at the burn out of each stage along the preliminary trajectory. Then the numerical software evaluates the optimal incidence angle $\alpha(t)$ for the stages with intent of maximizing the final stage velocity while satisfying the constraints on the final slope.

The optimization software employs a standard function for the evaluation of constrained minima of functionals in the following fashion: given the guess for the incidence law, the software calculates the trajectory and evaluates the cost function and the constraint violations, which are then used to generate a better guess for the incidence law. Now a Keplerian gravitational field is considered and the aerodynamic effects are taken into account. The aerodynamic coefficients are evaluated using the USAF Missile Datcom Software [10]. In particular the software was used to calculate the coefficients for a set of flight conditions at different Mach numbers and angles of attack; then the simulation software interpolates the aerodynamic coefficients. Generally the optimization of the last stage (liquid propellant engine) is performed using the thrust arc intervals as parameters and the optimization is performed in order to maximize the final mass. The constraint associated to the final stage is represented by the fulfillment of the terminal conditions related to the insertion into the desired orbit.Practical engineering constraints are also introduced, e.g. a minimum of one second of coasting times after the stages burn out in order to allow a safe stage separation. Moreover, additional coasting times are considered in order to obtain better performances.

Definitely, the optimization of the entire trajectory of a three stages rocket is made dependent on the selection of the following parameters: the final slopes of the first two stages γ_{1f}, γ_{2f}, the coasting intervals between the first and second stage and between the second and the third stage, the thrust arc intervals of the third stage. The algorithm proposed is rather effective despite its simplicity, and it has been applied to define suitable rocket configurations

4. Performances of EFA Launched Systems

The above analytic and numeric analysis has been applied to the European Fighter Aircraft (EFA), used as carrier aircraft of a rocket able to set same payload into orbit. According to the EFA characteristic and pilot experience, EFA is able to drop a rocket of 4070 Kg at velocity between 280/300 m/s, flight path angle of 40 deg and altitude equal to 12 Km. If a three stage rocket is considered, the above analysis gives for the first and second stages [3] :

$$n_{01} = 3.4 \ , \ t_{b_1} = 60\,s \ \ , \ \ n_{02} = 2.4 \ , \ t_{b_2} = 60\,s$$

and the following values of velocity, flight path angle and altitude at the burn out times are achieved:

Table 1. State variables at burn out times, according to the analytic solution

Time s	Velocity Km/s	Flight path angle deg	Altitude Km
0	0.30	40	12
60	3.625	18	56
120	4.873	11	120

Numerical refinement [8] gives the following parameters of the three stage rocket then

$$n_{01} = 3.39 \ , \quad t_{b_1} = 1.19 \ , \quad n_{03} = 2.56$$

Table 2. Main parameters of the EFA carried three stage rocket

	Stage 1	Stage 2	Stage 3
Length (m)	0.30	40	12
Diameters (m)	1	1	1
Mass (Kg)	3393	325	352
Propellant mass (Kg)	2958	270	237
Thrust (Kgf)	13804	1258	900
Specific Impulse (s)	280	280	300
Burn out time (s)	60	60	79

The trajectory of the rocket described by the parameters reported in Table 2 is integrated numerically taking into account the keplerian force field and the atmospheric drag with the above release conditions. The thrust direction is optimized according to the scheme presented in Section 3: the final slopes of the first two stages is fixed to the analytic values of the flight path angles presented on Table 1.

Table 3. Time schedule of the optimal trajectory

Time s	Vel. Km/s	F.p. angle deg	Altitude Km	Mass (Kg)	Events
0	0.300	40	12	4060	Separation
60	3.391	22.5	61.5	677	Stage 1 burn out
121	4.343	18.2	118	396	Stage 2 burn out
270	4.040	6.33	252	396	End of 1^{st} coasting
371	7.339	1.89	282	127	Stage 3 burn out 1
2366	6.842	0.003	700	127	End of 2^{nd} coasting
2368	6.988	0.00	700	120.6	Injection into orbit

The optimizer determines also the optimal coasting times between the second and third stage as well as the optimal thrust arc intervals of the third stage. The schedule of the optimal trajectory is presented on Table 3. The final mass into orbit is about 120 Kg, then the payload that the EFA

DOI: 10.1685/CSC09270

carried rocket can inject into an orbit of 700 Km is between 40 and 50 Kg (depending on the technological level of the liquid propelled engine of the third stage).

5. Final Remarks

Trajectory optimization is a rather lengthly process based on the preliminary knowledge of some parameters that must be refined by iterative numerical algorithms to achieve the convergence to an optimal solution. It is important to have some fast tool to get these preliminary results, mainly in the early phase of a new project when the past experiences are of little use and many different options must be investigated. In fact this is the case of launch systems released from an high performance aircraft.

Analytical formulas have been used to get first guess conditions of optimal trajectories of air-dropped rocket. These formulas are based on a very simple model: the analytical trajectory is obtained in the hypothesis of gravity turn guidance and no aerodynamic effects. These effects are reduced at the altitudes where an air-dropped rocket is released and although the gravity turn guidance is not optimal, the analytic formulas are able to provide a preliminary rocket design and rocket optimal trajectory. Namely the state variables reported in Tables 1 and 3 show that the analytic formulas provide with values that are good enough to get numerical convergence to the actual optimal trajectory.

The method presented here has been applied for different launch systems, based on different carrier aircraft and different dropping conditions. In particular it is shown that EFA launched rocket can inject up to 50 Km into circular orbits of 700 Km of altitude.

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