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PROMOTE CONNECTIONS BETWEEN MATHEMATICS DRAWING AND HISTORY OF ART IN HIGH SCHOOL THROUGH A STEM APPROACH

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ABSTRACT. The paper is part of the debate surrounding the possibilities for advancing the role of Mathematics within STEM education through interdisciplinary approaches. It provides an example of how high school students can engage in tasks of modelling (interdisciplinary approach) directed to promoting connections between Mathematics and 'Drawing and History of art'. Specifically, the paper describes a teaching practice focused on the theoretical and applicative implications of the process of interpretation and construction of the Vignola's geometric method for laying out the Ionic volute. The study of the method makes use of drawing as a formal analysis tool in order to make the students grasp the sense of the graphic representation synthesized by the geometric model. The intuition of the geometric genesis of the model is then verified on the computer with the use of the GeoGebra software.

1. Introduction

Most of the frameworks for 21st century skills refer to skills such as creativity, critical thinking, and problem-solving. All these key competencies and the dynamic combination of knowledge, skills and attitudes, can be particularly developed by an interdisciplinary approach with STEM subjects' (Science, Technology, Mathematics and Engineering) activities in the classroom, which underlines the importance of designing educational activities that facilitate deep learning to enhance these 21st century skills (Voogt and Pareja Roblin 2012). An integrated STEM approach is a valuable tool to develop not only for a deeper science literacy but also capable citizens who personally and professionally make informed decisions in their daily lives based on a critical review of given information and who have the capacity to enter the future job market with its growing request for STEM careers. The European Council recognizes the central role of technologies, entrepreneurial, social and civic skills which allow citizens to be able to adapt to changes in our society and to develop the higher and more complex skills necessary to guide creativity and innovation (Communication COM / 2015/0614) [1].

[1]Retrieved from https://eur-lex.europa.eu/re-source.html?uri=cellar:8a8ef5e8-99a0-11e5-b3b7-01aa75ed71a1.0012.02/DOC_1&format=PDF

In Italy, initiatives aimed at STEM education (D.M. 851/2017)[2] have also been launched to encourage female enrolment in scientific degrees. This is an important signal aimed at strengthening the process of innovative teaching, but which still seems not to reach sufficient strength due to resistance because of a school still bound to a traditional structure.

A structure that has never changed at the system level, but only in single school realities, which, however, have not managed to create a critical mass for a real change in the conception of the school curriculum. STEM integration is a curricular approach that combines the concepts of STEM in an interdisciplinary teaching approach (Wang *et al.* 2011). The goal of integrated STEM education is to be 'a holistic approach that links the disciplines so learning becomes connected, focused, meaningful, and relevant to learners' (Smith and Karr - Kidwell 2000, p. 22). The interdisciplinary approach offers students one of the best opportunities to experience learning in a real-word situation, rather than to learn bits and pieces and then to have to assimilate them at a later time (Tsupros *et al.* 2009).

In mathematics education, the integrated approaches to STEM education can contribute to preparing individuals better for twenty-first century challenges, including dealing with socio-scientific issues and responsible research and innovation in the search for societies that are ethically acceptable, socially desirable, and sustainable (Maass *et al.* 2019).

This approach involves a review of the teaching of mathematics in the classroom: not as rote procedure, but as the codification of experiences of both making sense and sense making through various practices including problem-solving, reasoning, communicating, and mathematical modelling. The implementation of this practice in the classroom allows the teacher to put the emphasis on higher registers of language (functional, algebraic, geometric, cultural, technical-IT) in an active and tangible way, thus allowing the student to generate autonomous reflections suitable for an effective internalization of the contents and for the construction of scientific thought.

This paper presents an interdisciplinary approach – based on modelling – for advancing the role of Mathematics within STEM education. Specifically, it is a teaching practice aimed to promote connections between Mathematics and 'Drawing and History of Art', disciplines which characterize the cultural, educational and professional profile of the scientific high school of the Italian school. The content integration concerns the geometric method which was conceived by architect Jacopo Barozzi and is called Vignola [3], for tracking the Ionic volute. The study of the method is carried out by reading and interpreting the original text, using drawing as a formal analysis tool to deduce the geometric genesis of the model. Between theory and practice, the study of the method is carried out by letting students experience the 'doing' of it in a digital choreography in which the geometric model is checked on the computer with the GeoGebra software. This is an interdisciplinary experience during which the students learn concepts and skill from various disciplines that are tightly linked so as to deepen their knowledge and skills, and at the same time discover maths concepts and make generalizations, thus developing and promoting a different kind of mathematical thinking. The paper is structured as follows. Section 2 presents the theoretical

[2]Decreto Ministeriale 851/2017- art. 35. Piano di ricerca e formazione per una didattica innovativa delle STEM Retrieved from: https://www.pianoricercaeformazione.it/ per le classi di concorso della scuola secondaria https://www.istruzione.it/scuola_digitale/alle-gati/2019/D.M.%20N.%201008%20DEL%2021.12.217.pdf

[3]Jacopo Barozzi da Vignola, often simply called Vignola, (1507 – 1573), was one of the great Italian architects of 16th century Mannerism.

framework used. Section 3 covers all steps of the teaching practice; and finally, in section 4 conclusions and recommendations are suggested.

2. Theoretical framework

Although it is commonly acknowledged that mathematics is foundational to STEM, in practice mathematics is usually related to STEM education at a distance (English 2016b).

Holding the conception of mathematics as a product does not support integrating mathematics with other STEM disciplines, as mathematics can be perceived simply as a set of tools for these disciplines. At the same time, mathematics and science have often proceeded along parallel tracks, with mathematics focused on "problem solving" while science has focused on "inquiry" (Li and Schoenfeld 2019). To better connect mathematics and other disciplines in STEM, we should focus on the ideas and thinking development in mathematics advocating the different ways of sense making (Schoenfeld 1992).

The making-sense stance, as termed by McCallum 2018, generally is not practiced in the classroom because it is considered pertinent only to expert mathematicians' practices. Sense making and making sense are two different concepts. Where sense making emphasizes the process of making sense of what is being learned, making sense emphasizes the process of making mathematics make sense (Li and Schoenfeld 2019).
Making sense highlights the importance for students to experience mathematics through creating, designing, developing, and connecting mathematical ideas.
Emphasizing both sense making and making sense in mathematics education opens opportunities for connections with similar practices in other STEM disciplines because it is often combined with reflections in engineering (Kilgore *et al.* 2013), also in the context of using technology (e.g., (Dick and Hollebrands 2011)). Conceiving mathematics as making sense help promote conceptual changes in classroom practice so as to value idea generation and design activity.
Connections generated from such a shift will support teaching and learning not only in individual STEM disciplines, but also in integrated STEM education. At the same time, although STEM education as a commonly recognized field does not have a long history (Li 2014, 2018), its rapid development can be seen as an aid to introducing ideas for exploring how mathematics can be taught and learned. While it is widely acknowledged that mathematics underpins all other STEM disciplines, there is clear evidence it plays an understated role in integrated STEM education (e.g., Fitzallen (2015) and English (2016a)). In this scenario, it is necessary to promote advancing the role of Mathematics within the field of STEM in education activities. Maass et al. (2019) (Maass *et al.* 2019), starting from the nature of STEM education, suggest three interdisciplinary approach possibilities:

(1) the acquisition of twenty-first century skills;
(2) meaningful inclusion of mathematical modelling in school education;
(3) education for responsible citizenship.

The purpose of this paper is to address the second of these interdisciplinary approaches to reinforce the perception that Mathematics play a major role in all STEM fields. Coming from an interdisciplinary nature of mathematical modelling, the authors describe an approach to the design and implementation of a teaching practice that connects Mathematics to 'Drawing and History of Art'.

In Italian higher schools the connection of Mathematics with other disciplines is often taken as a given truth. An example is given in the educational path of scientific high schools where the connection between 'Drawing and History of Art' and Mathematics often is considered too difficult for the students to experience, except in a few standard cases mentioned in most textbooks. It is not easy to grasp the link between the History of Art and Drawing as the latter, despite constituting the fundamental form of visual communication, serves not only to depict what exists, but also to schematize an idea, to sketch a work quickly and succinctly, and also detailed and precise. The graphic/geometric drawing assumes the role of language of a descriptive thought of forms and figurations that must be used by the students to learn to systematically and historically understand the physical environment in which they live. For example, the link between geometry and architecture is so close that one cannot distinguish between one art and the other, that is, between the art of thought that has no body, and the art of the serious work that gives sensible shape to the idea. In fact, if the deductions of the geometric reasoning are completely abstract, it is also true that the same deductions can be verified experimentally through the drawing and, with the current state of affairs, also through digital representation, with a degree of reliability of the experimental verification that depends on the accuracy of the system used for the representation. This graphic computerized verification is similar to the physical construction, even though it takes place in a virtual space. The geometric construction can indeed rise to a method of existential demonstration, in the sense that the possibility of constructing a geometric figure assures us of its existence (Loria 1935). Teaching must adapt to this function, therefore the teaching of 'Drawing and History of Art' must set inductive processes in motion, stimulate the acquisition of work and research methodologies in close connection with the other disciplines, especially with Mathematics.

Exploring the Maths and Art connection, the strategic use of technological tools such as a Dynamic Geometry Environment (DGE) is of particular importance both for simulations –understood as experimenting with models–, and because it supports students' mathematical thinking and discourse (Drijvers *et al.* 2016; Serpe 2018; Serpe and Frassia 2020). The simulation with the computer can be seen as an integral part of didactical engineering. According to Balacheff and Kaput (1996), the interaction between a learner and a computer is based on a symbolic interpretation and computation of the learner input, and the feedback of the environment is provided in the proper register allowing its reading as a mathematical phenomenon (Balacheff and Kaput 1996, p. 479). For example, the GeoGebra software, thanks to its dynamic potential, can be used as a tool for reading and interpreting a graphic source. These potentials –combining geometry, algebra, graphics and more in a single user-friendly application– are found to be optimal for investigating the intrinsic complexity of architectural treatises as they help mediate between text and image. The mediation is often in the form of externalized visual models, where the interpretation and use of models is common and often a central practice in STEM domains. Mathematical modelling provides students with an opportunity to experience how models come to be and interrogate the trade-offs involved in developing a mathematical model, including assessing the limitations and strengths of different model. Mathematical modelling applications provide to develop students' analytical thinking abilities, problem-solving abilities, and the qualifications needed in this technology-based information (Stillman *et al.* 2015; Frassia and Serpe 2017; Serpe 2018).

The present work fits into this theoretical framework and puts forward an example of teaching practice to connected Mathematics and 'Drawing and History of Art'. Coming from a STEM integration perspective, the teaching practice proposed on the simplified 'Engineering design process' (Figure 1) (Hynes *et al.* 2011).

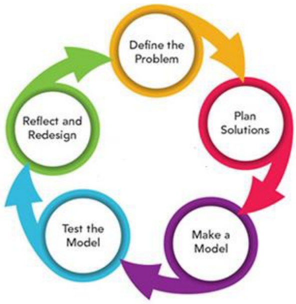


FIGURE 1. Engineering design process (Hynes *et al.* 2011).

By adopting this paradigm's lines, students do not rely upon rigid thinking, but this provokes creative and outside-the-box thinking. In this way the teacher provides opportunities to play with knowledge, generate ideas, share and critique, and ultimately strive to understand the effectiveness of one's ideas. Having opportunities to engage in processes that afford such tinkering may also help students expand their repertoire of epistemic resources situated within the context of classroom-based problem-solving activities. The Engineering design process consists of five steps and the details involved in each step are listed in Table 1.

TABLE 1. The five steps of Engineering design process.

Step	Description
Define the Problem	What is the problem? How have others approached it? What are your constraints?
Plan Solution	What are some solutions? Brainstorm ideas. Identification of key variables and /or mathematical objects.
Make a Model	Draw a diagram. Make lists of materials you will need. Definition of relationships between variables and /or mathematical objects.
Test the Model	Follow your plan and create the model. Translation of the identified relationships.
Reflect and Redesign	Works? Modify your design to make it better. Analysis and interpretation of the results obtained. Possible refinement of the model and related understanding through an iterative process.

From Table 1, it is clear that the application of the fourth step (Test the model) previews the simulation of model of Ionic volute by Vignola in the DGE GeoGebra. Of course, computer modelling plays an important role in the engineering design loop: the students must create the algorithm to implement. An implemented algorithm is, on the on hand, an intellectual gesture, and, on the other, a functioning system that incorporates in its structure the material assumptions about perception, decision and communication.

3. Teaching practice implementation with STEM approach

The teacher starts from the historical problem in its broad outlines in order to restore to ideas their original character of discovery and novelty and to contribute, in an essential way, to building around the concepts' meanings, applications and openings that often remain hidden in the subsequent formal arrangement. The subsequent phases involve the study of one of the geometric methods proposed by Vignola for tracing the Ionic volute using drawing as a formal analysis tool between past and present: drawing with ruler, square and compass and, subsequently, on the computer with GeoGebra software. The computer reinterpretation allows the construction of the meaning of the graphic representation through the model (polycentric spiral) that highlights and communicates the geometric synthesis. The mathematical model contains within itself an expressive and evocative potential from a metric and formal point of view as it allows us to eliminate specific details, to simplify by highlighting the aspects of greatest interest.

3.1. Define the problem – The Ionic volute. The class is divided into groups and each group of students is called to examine the capital of the Ionic column (Figure 2), a topic already addressed in the field of 'Drawing and History of Art'. Students are encouraged, by the mathematics teacher, to carefully revisit this architectural element: observe the form and, therefore, derive an initial characterization. Through a "provoked" and guided conversation, the teacher will highlight how the drawing is a figuration made on the basis of a more or less complex path to convey information relating to existing or imaginary objects, concepts and emotions. Therefore, the drawing constitutes a tool for investigation, study and design not only in the world of art but also in that of science and technology.

FIGURE 2. Ionic capital, north porch of the Erechtheion, 421-407 BCE, Acropolis, Athens.

The students confronted with this empirical situation will notice that the cap of the Ionic column, unlike the other two architectural orders (Doric and Corinthian), has two curls on its sides, called volute [4] that recall the shape of a spiral. Following this, the students, also helped by the 'Drawing and History of Art' teacher, will carry out appropriate research in

[4]from the Latin *volvere* which means to turn, wrap.

order to be able to outline a more complete cognitive picture of the architectural element in question (Figure 3).

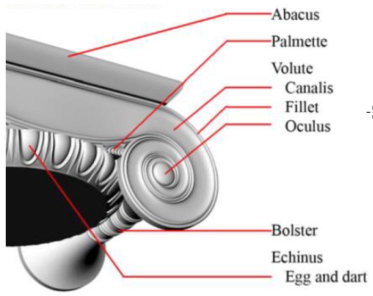


FIGURE 3. Anatomy of the Ionic capital.

From the research various points will emerge:

- how, since the appearance of the Ionic order (7th century BC), the spiral shape was clearly recognized in the two volutes that characterized the capital;
- the historical question around the form of this architectural element starting from the moment in which the drawing becomes a tool for the faithful representation of forms, and the practice of using orthogonal projection spreads (Recht 2001, p. 135).

Through appropriate insights, students will have the opportunity to discover that the debate began between the fourteenth and fifteenth centuries, and originated from the difficulties of interpretation of a short passage of the third book of the treatise *De Architectura* by Vitruvius[5] (Vitruvius 1999) aimed at the development of a method for tracing the volute that was then applicable in the building practice of the fifteenth century, and thus allowed a generalized use of the Ionic order. In essence, drawing the Ionic volute involves distinguishing a graphic method for laying out a curve, which from a smaller circle with a given dimension one makes a certain number of turns towards a larger circle, also of a given dimension, which is concentric with respect to the first. This represents the mathematical definition of a spiral segment (Collins 2009, p. 529)
The problem that emerged arouses in the students a taste for investigation in the two opposite senses: towards the future to which, naturally, they feel attracted and towards the past whose fascination they undoubtedly feel.

On this basis, the synergistic collaboration of the two teachers is fundamental in order to prepare a simplified reconstruction of the historical development of the question starting, precisely, from the writings of Vitruvius which provide the description of some very clear basic rules, but which can be difficult to interpret as far as the actual tracing of the volute is concerned, especially beyond the firstlap around the center. In fact, Vitruvius [6], in the representative scheme, does not provide concrete measures, but only proportional

[5]Marcus Vitruvius Pollio (c. 80-70 BC – after c. 15 BC) is the author of *De Architectura, libri decem*, known today as *The Ten Books on Architecture*, a treatise written in Latin, dedicated to the emperor Augustus. This work is the only surviving major book on architecture from classical antiquity.

[6]De Architectura, book III, 256.

relationships, indicating that the volute is traced by means of quarters of a circle, using the compass starting from the outermost point, until it reaches the eye. The Vitruvian text, therefore, does not specify how many quarters of a circle are needed to complete the volute, nor does it indicate the position of their centers, nor does it specify how many laps the volute must complete in total; the resulting spiral is mathematically inexact and, adopted as intended, appears awkward.

3.2. Plan solution – The root of the Ionic volute: the spiral. At this point, the students are definitely faced with a mathematical object. The drawing of the Ionic volute comes from the shape of the two-dimensional spiral, a flat curve that from a smaller circle, of given dimensions, unfolds in a certain number of laps towards a lager circle, concentric with the first, and of given dimensions as well.
In other words, an infinitely extended plane curve generated by a point that fulfils a composed motion, of rotation around a center and translation along a ray, outgoing from the center. There are several families of spirals, but only a few have practical applications in architecture, such as the spiral of Archimedes [7], logarithmic spiral and polycentric spirals. The spiral of Archimedes, exhibited for the first time in his work *On Spirals*– a treatise andressed to the mathematician Dositeo of Alessandria - is generated by a point which moves at a uniform speed on a ray, which, in turn, rotates uniformly around an 0 point, with a constant angular speed and a constant pitch (Archimedes 2009).
The Archimedean spiral (Figure 4) is describe in polar coordinates by

$$r = a\theta \qquad a > 0 \tag{1}$$

where a is the constant coefficient of proportionality (the parameter of the Archimedes spiral), r is the length of the radius from the center, or beginning, of the spiral, and θ is the angular position (amount of rotation) of the radius.

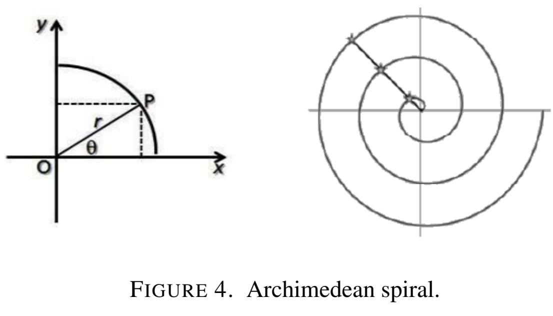


FIGURE 4. Archimedean spiral.

The distance between two consecutive turns is constant, and it is equal to $2\pi r$, if the angle θ is measured in radiant.
This spiral has two branches, for starting the angle $\theta > 0$ or $\theta < 0$. The equation (1) can be converted in the parametric form using the relationship between polar coordinates and Cartesian coordinates to solve for x and y. If the point P has Cartesian coordinates (x, y), and polar coordinates (r, θ) with the angle chosen from $[-2\pi, 2\pi]$, then from this, we have:

[7] Archimedes (born c.287BCE, Syracuse – died 212/211 BCE, Syracuse), the most-famous mathematician and inventor in ancient Greece.

$$cos\ \theta = \frac{x}{r} \qquad sin\ \theta = \frac{y}{r}$$

So

$$x = r\ cos\ \theta \qquad y = r\ \sin\theta$$

From this is follows that the parametric equations of an Archimedean spiral are:

$$\begin{cases} x = ar\ cos\theta \\ y = ar\ sin\theta \end{cases}$$

The logarithmic spiral also known as the equiangular spiral or growth spiral (Figure 5), instead, is characterised by a pitch that rises from the eye of the volute, moving outwards, according to a logarithmic law.

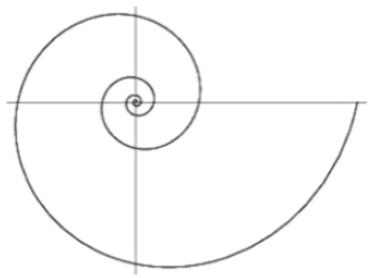


FIGURE 5. Logarithm spiral.

In the logarithm spiral, the point *P* moves in a straight motion on a ray of origin *O* so that the logarithm of its distance from pole *O* is proportional to the angle of rotation of the same ray around the pole. The equation in polar form is

$$log\ r = a\theta \tag{2}$$

where *a* is constant. Rewriting the equation (2) in exponential form gives

$$r = e^{a\theta}$$

where *e* is Euler's number, the constant *a* is the rate of increase of the spiral. The sign of *a* determines the direction of rotation of the spiral.

A logarithmic spiral is a spiral such that the angle between the tangent and the radius vector is the same for all points of the spiral. It is can be distinguished from the Archimedean spiral by the fact that the distances between the turnings of a logarithmic spiral increase in geometric progression, while in an Archimedean spiral these distances are constant.

The polycentric spiral is a curve obtained by linking circular arcs with different center and radius, having the same tangent in their point of contact. For this essential condition to be verified, it is obviously necessary that the centers of the two arcs are aligned with the point of contact. In this way the common tangent will be perpendicular to the line that contains the centers and the connection.

3.3. Make a model – Geometric method of Vignola's for laying out the Ionic volute. From the perspective of historical reconstruction, the students should continue their investigation into the identification of a graphical method for tracing the Ionic volute. They will discover that in the architectural treatises compiled since the Renaissance there are many different solutions relating to the tracing of the spiral; these are geometric methods that use the compass as the fundamental tool and the arc of circumference is the basis for tracing the volute. Unlike the general proportions of the capital, for the tracing of the volute, the treatises deviated more and more from the indications given by Vitruvius, to arrive at the architect Jacobo Barozzi (known as Vignola), and entirely original solutions. In fact, Vignola in his treatise *Rule of the Five Orders of Architecture*[8] (Barozzi da Vignola 1562) proposes two different geometric methods of the volute, the first which simplifies the Giuseppe Salviati's[9] construction and the second which takes up the model of Guillame Philandrier[10]. Both methods, although very different, however, lead to very similar results. Particularly, the first method starts from an entirely new idea, obtained by discretizing the continuous form of the volute in a series of arcs of circumference accurately connected to each other so as to ensure the smoothness of the curve.
The construction of the geometric method was set in the eye of the spiral (Figure 6).

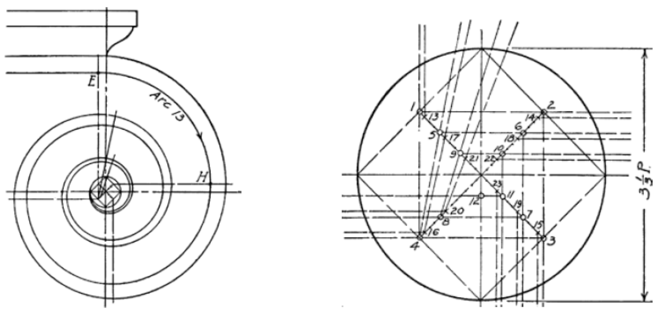


FIGURE 6. Graphic analysis of Vignola's Ionic volute. On the right, detail of eye.

It is now a question of studying the aforementioned geometric method in detail. The students now have to carry out some research, via the Internet, aimed at finding the original text file of the treatise *Rule of the Five Orders of Architecture* (Barozzi da Vignola 1562) containing the instructions for tracing the Ionic volute. Once the file has been downloaded, Table XX, Book I, Chap. XVI (Barozzi da Vignola 1562) is extrapolated, which provides the practical prescriptions of the method (Figure 7).
Table XX will be observed by students with the help of teachers; therefore, the vertical composition will emerge representing the design of the Ionic volute accompanied by concise

[8]Treatise, drawn up in 1562, which synthesizes and codifies the classical architectural lexicon, also proposing a calculation system to facilitate the task of the builders in designing and carrying out their works according to the five architectural orders.

[9]Giuseppe Porta (1520–1575), also known as Salviati, was an Italian painter of the late-Renaissance period, active mostly in Venice.

[10]Guillaume Philandrier (1505-1563), is a humanist and author of several commentaries on works from Antiquity. Especially, his comments on *De Architectura* by Vitruvius, earned him fame.

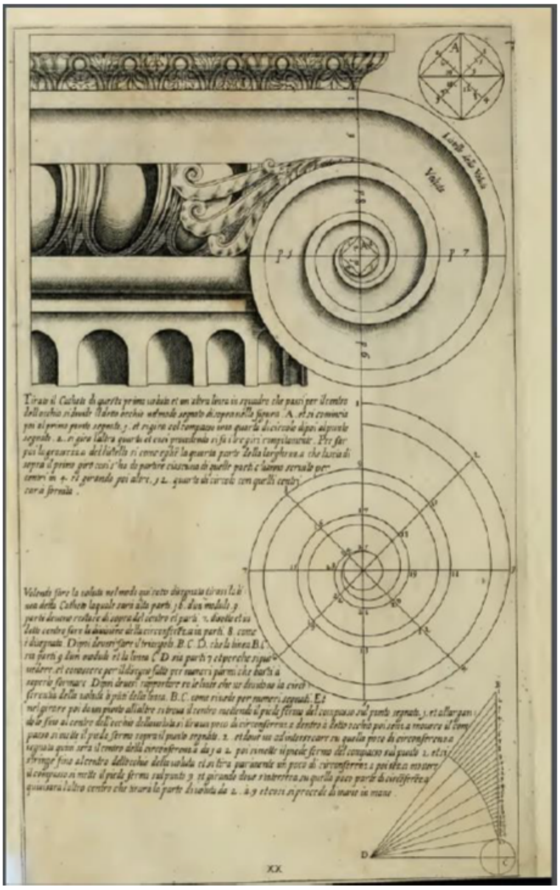


FIGURE 7. Table XX - *Rule of the Five Orders of Architecture*, by Barozzi da Vignola (1562).

lines of explanatory text on the 'rule' chosen as the solution for the tracing. The 'rule' is described in the following text (Barozzi da Vignola 1562):

> "*Having drawn the cathetus* [11] *of this first volute and another square line*[12] *that passes through the center of the eye, the said eye is divided in the manner indicated above in figure A, and then begins at the first point marked 1, and turns with the completion of a fourth of a circle, then at the point marked 2, the other fourth is turned and thus proceeding, the three turns are completed correctly. Then to make the thickness of the strip* [13] *as it is a quarter of the width that leaves from the top of the first round,*

[11]From the Greek κὰθετος perpendiculare line.

[12]At right angles to the cathesus.

[13]The volute's flat and raised edge.

you must divide each of those parts we have used as centers in 4 and then turning over, 12 quarts of a circle will be provided with those centers."

Table XX, although accompanied by illustrations, is described in a classical language which is not immediate for the students. This problem of a terminological nature can be overcome with the help of teachers who will facilitate the interpretation of the text by matching textual and figurative data. With a guided conversation, the reading and interpretation of the text will be made clearer and more accessible by channelling the language into the specific lexicon of geometry. Students will thus be facilitated in the exploration and understanding of the tracing technique used by Vignola; at the same time, they are stimulated to reflect on the reasons for drawing as a study tool and approach to knowledge. The close and synergistic collaboration of teachers at this stage of the study path is essential to gradually lead students to the interpretation of the graphic analysis of Vignola's 'rule'. In fact, graphic analysis is a powerful tool that allows you to break down the structure of a design to search for elementary geometric shapes.

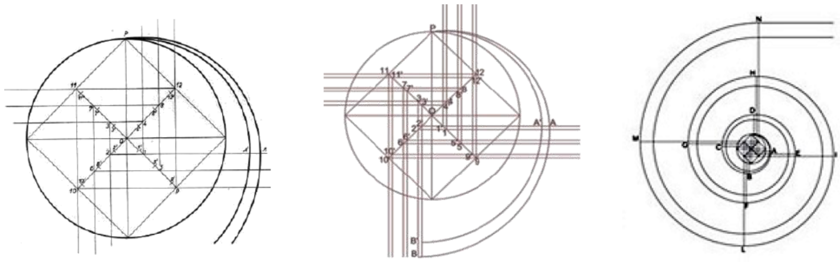
Once the geometry has been identified, it is now possible to describe the semantic connections of the tracing method through the following instructions:

- draw two equal perpendicular segments that intersect at their midpoint O, where the compass is centred to draw a circumference with a radius equal to half of the segments traced.
- Let P be a point on the circumference, extreme of one of the diameters, from which the spiral will start; the two diameters of the circumference correspond to the diagonals of the inscribed square.
- Divide its medians into six equal parts, numbering them clockwise or anticlockwise, according to the desired development of the spiral (Figure 8a).
- From the dividing points draw four series of parallels to the diameters of the circumference, extending them to the outside (Figure 8b).
- With center at point 1 and radius equal to distance P1 describe the arc PA, which will be joined with the next one of center 2 and radius equal to distance A2 up to point B.
- Continue centring in 3 with radius B3 up to point C and repeat the procedure up to point N, centring in all points up to the twelfth.
- The internal profile of the volute is obtained in a similar way, but based on the centers 1', 2', 3'... 12', which are located at a quarter of the distance between the previous points, towards the inside (Figure 8c).

The mathematical model defined by the instructions above is a polycentric spiral with only two non-polycentric fittings, due to misalignment of the point of contact with the centers of the arcs to fillet. In all there are 12 circular arcs, the centers of which are located at the top of three concentric squares, the largest of which has a side equal to the radius of the eye.

The method, of considerable formal elegance, makes explicit reference to Euclid's Elements using, in particular, XI and XIII propositions of the third book [14] and the V

[14]Prop. XI: If two circles touch one another internally, and their centers are taken; then the straight line joining their centers, being produced, falls on the point of contact of the circles.
Prop. XIII: A circle does not touch another circle at more than one point whether it touches it internally or externally.



(a) (b) (c)

FIGURE 8. Subsequent tracking steps of Vignola's geometric method.

proposition of the fourth book [15] (Heath and Heiberg 1908). Indeed, the method has the compass as the instrument of the fundamental drawing, and the arc of the circle as the basis of the laying out of the volute. Due to its geometric nature, the method pervasively occupied architectural treatises until the early twentieth century.

3.4. Test of model – Computer modelling with GeoGebra of Vignola's Ionic volute. In the previous phase, the students discovered, in Vignola's graphic corpus, a mathematical heritage that can become a new tool for design verification in a more current perspective, namely the digital one. With the use of the GeoGebra software, the steps that led to the geometric genesis of the shape can be redesigned, in a virtual computer space, simulating the construction process. This operation makes it possible to build the geometric model of the Ionic volute, making any kind of modification possible: the virtual model thus becomes an active corrector in the design phase.
Computer modelling is governed by a mathematical algorithm that translates the tracing methods into analytical terms according to a modus operandi that traces the history of construction technique. The creation of the algorithm requires a careful re-reading of the geometric aspects that characterize the method and the subsequent implementation on the computer helps students to reflect on the way in which all these aspects are held together in the humble container of the algorithm (Serpe 2020).

The creation of the algorithm constitutes an important and delicate moment as students have to design that "finite succession of steps" that allow the computer to simulate the geometric construction, or to highlight the need for further work. It is not a simple operation because it presents obstacles inherent in the procedure in which abstract and formalizable objects must be manipulated (Costabile and Serpe 2010).

[15]Prop. V: About a given triangle to circumscribe a circle.

3.4.1. *Algorithm.*
The steps of the solving algorithm are as follows:

1. Fix a Cartesian coordinate system in the plane xOy;
2. draw two congruent segments of length l and perpendicular, which meet in O, mid-point of the drawn segments;
3. trace the circumference with center O and radius $r = l/2$;
4. draw the square inscribed on the circumference/circle with diagonal length $2r$;
5. construct the medians of the square and divide them into six congruent parts;
6. from the determined points draw four sets of straight lines parallel to the diameters of the circumference, extending them outwards;
7. cycle for the construction of the internal and external volute:
   a) construction of arcs with center points constructed on the diagonals of the square for the internal volute;
   b) construction of the arcs with center points obtained from the points on the diagonals of the square at a distance of 1/4 of the radius of the circumference inwards.

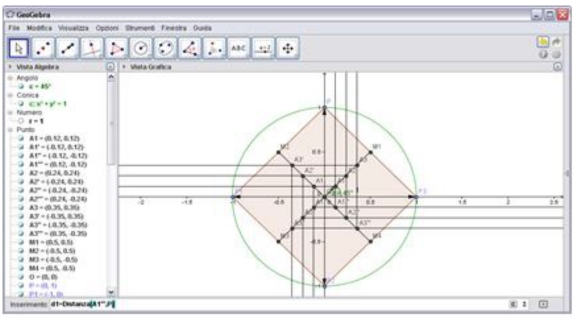


FIGURE 9. Output of the initial construction of the Ionic volute.

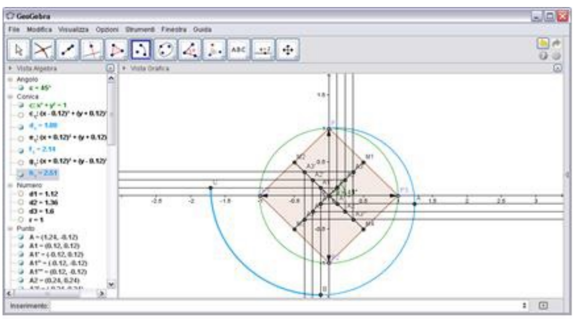


FIGURE 10. Cycle output for external construction of the volutes.

The internal part of the spiral is obtained in the same way, considering the centers at one quarter of the distance between the previous points.

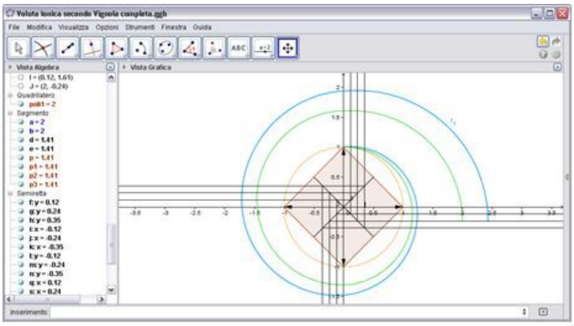


FIGURE 11. Cycle output for internal and external construction of the volutes.

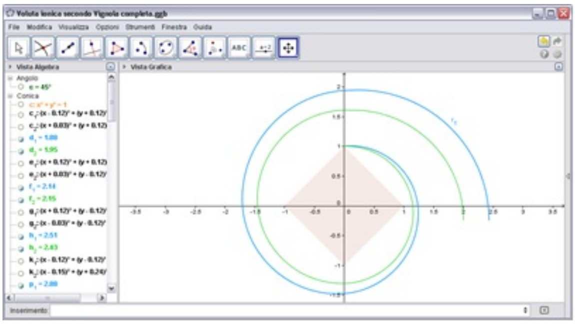


FIGURE 12. Cycle output for internal and external construction of the volutes with hidden steps.

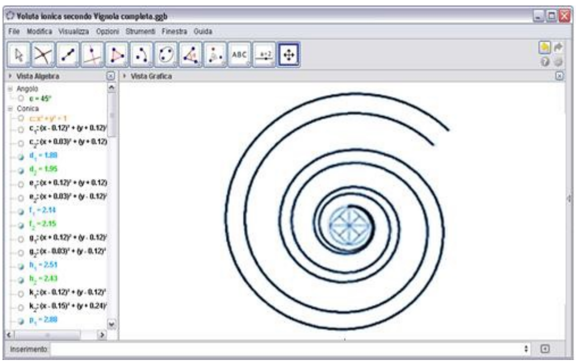


FIGURE 13. Final output of the Ionic volute.

3.5. Reflect and redesign – Comparison between the mathematical model and the architectural element. The geometric method develops around the enveloping movement of the spiral, root of the Ionic volute. In this step, students have the opportunity to reflect on the difference between the graphic operation (to draw) and the theoretical and abstract thought on the form and architecture. The constructive process of computer modelling with GeoGebra software represents a valid aid along the way as they progress in the same way from what is predefined to what is made by the user. The students, through a constantly evolving process, have the possibility to manipulate mathematical objects and rediscover some mathematical terms linked to geometrical construction.

The process culminates with the comparison between the mathematical model and the shape of the architectural element. Any mismatch represents an opportunity to retrace one's steps: students are asked to retrace the modelling process carried out to identify errors and / or imperfections.

The focus on redesigning aims to improve the prototype up to the realization of a final product that meets all the requirements and criteria. Though the algorithm seems to imply a set path, it is important to remember that the students are constantly evaluating and testing their ideas, repeating steps as necessary and sometimes even restarting from the beginning. Occasionally the original idea will have some initial overlooked flaw or a different approach may become apparent through work on the challenge.

According to Monge (1798) the representation of the figure and its geometric genesis "offers continuous examples of the passage from the known to the unknown", and, that is, continuous suggestions and invitations to explore what abstract thought had not foreseen (Monge 1798). These characteristics make it possible to recognize geometry as an abstract architectural exercise. Indeed, the geometric construction helps the students to begin just this complex path which starts with the simple and evident and moves on to the complex and 'non-evident' in a tangible, critical and rigorous way.

In this way, the students learned through the process of making which included observing, recognizing patterns, practicing, studio techniques, creative thinking skills, and collaboration and communication among peer.

4. Conclusions

The teaching practice shown provides the real opportunity to see how mathematics intervenes in architecture on at least three levels: as a language, as a representation and as a structure. In particular, it highlights how among the reasons of architecture, the geometric and numerical ones provide the elements that articulate the logic of the forms, allowing them to be controlled. The graphic analysis of the volute leads students to immerse themselves in a historical dimension that highlights the deep bond that unites geometry and architecture: both are arts of building.

Architecture gives physical body to geometric abstractions, while geometry controls their ideal shapes in the design phase. Here it is evident how the abstract form, constructed and controlled geometrically, has been translated into a skilful stone structure with the aim of suggesting and enhancing the perfection of the spiral, assumed in the design idea. Then, the reproduction of the design of the scroll on the computer leads the students to think about not only mathematical concepts, but to reflect and question themselves on the relationships existing between mathematics, drawing, and computer science. In fact, the computer simulation requires the formalization of a series of simple instructions that leads students to investigate the connections and relationships between the elements in question, thus checking the accuracy of the line (s) previously drawn. The use of the GeoGebra

software used for graphic reconstruction offers students a decidedly engaging approach aimed at enhancing conceptualization and abstraction skills.
The whole process has a great educational value, because it allows for a more thoughtful formalization where the same demonstrations, properties of geometry already addressed, occur, exploiting its investigative and intuition support potential (Serpe and Frassia 2018).

This teaching practice is in no way limited but constitutes a starting point for similar and subsequent work to be designed and implemented in classrooms because it is based on the ability to show students not only a different application of mathematics, but also a deep connection with other disciplines.

The STEM approach used helps students to understand how mathematics is not just a tool for analytical investigation, but can more deeply underlie the developments of architectural design, and beyond. In fact, the implemented engineering design process constitutes an example in this sense: classroom experience aimed at identifying the basic geometries useful for a description of architectural complexity, starting precisely from a graphic source.

The STEM approach helps to give mathematics a different image, that is, more interdisciplinary and therefore more formative not only on a purely cognitive but also informative level. In this way, students really have the opportunity to broaden cultural horizons and develop both skills in the use of mathematical methods, tools and models in different situations, and aptitudes to re-examine the knowledge gradually acquired. Ultimately, architecture and mathematics are disciplines that have always cultivated reciprocal interests, finding in mutual complementarity continuous opportunities for growth and comparison. In the current educational scenario, privileging the STEM approach means highlighting how mathematics has played and continues to play the role of great mediator between science and technology as well as between science and art, revealing itself as subject-object and instrument -intention of the architectural-tectonic production.

References

Archimedes (2009). “On Spirals”. In: *The Works of Archimedes: Edited in Modern Notation with Introductory Chapters*. Ed. by T. L. Heath. Cambridge Library Collection - Mathematics. Cambridge: Cambridge University Press, pp. 151–188. DOI: 10.1017/CBO9780511695124.014.

Balacheff, N. and Kaput, J. J. (1996). “Computer - Based Learning Environments in Mathematics”. In: *International Handbook of Mathematics Education. Kluwer International Handbooks of Education*. Ed. by A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, and L. C. Springer, Dordrecht, pp. 469–501. DOI: 10.1007/978-94-009-1465-0_14.

Barozzi da Vignola, J. (1562). “Rule of the Five Orders of Architecture”. Retrieved from: http://architectura.cesr.univ-tours.fr/Traite/Auteur/Vignole.asp?param= (Figure Tavola XX).

Collins, H. (2009). *Dictionary Mathematics*. Glasgow: Harper Collins.

Costabile, F. and Serpe, A. (2010). “Archimedes in secondary schools: a teaching proposal for the math curriculum”. In: *The Genius of Archimedes – 23 Centuries of Influence on Mathematics, Science and Engineering. History of Mechanism and Machine Science vol.11*. Ed. by S. Paipetis and M. Ceccarelli. Springer, Dordrecht, pp. 479–491. DOI: 1007/978-90-481-9091-1_36.

Dick, T. P. and Hollebrands, K. F. (2011). *Focus on high school mathematics: Technology to support reasoning and sense making*. Reston, VA: NCTM.

Drijvers, P., Ball, L., Barzel, B., Heid, M., Cao, Y., and Maschietto, M. (2016). *Uses of Technology in Lower Secondary Mathematics Education*. ITS. Springer, Cham. DOI: 10.1007/978-3-319-33666-4.

English, L. D. (2016a). "Advancing mathematics education research within a STEM environment". In: *Research in mathematics education in Australasia 2012–2015*. Ed. by K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, and K. Fry. Singapore: Singapore: Springer, pp. 353–371. DOI: 10.1007/978-981-10-1419-2_17.

English, L. D. (2016b). "STEM education K-12: perspectives on integration". *International Journal of STEM Education* **3**(1), 1–8. DOI: 10.1186/s40594-016-0036-1.

Fitzallen, N. (2015). "STEM education: What does mathematics have to offer?" In: *Mathematics education in the margins. Proceedings of the 38th annual confer-ence of the Mathematics Education Research Group of Australasia*. Ed. by M. Marshman. Sydney: Sydney: MERGA, pp. 237–244. DOI: 10.1007/978-981-10-1419-2_17.

Frassia, M. G. and Serpe, A. (2017). "Learning Geometry Through Mathematical Modelling: An Example With Geogebra". *Turkish Online Journal of Educational Technology* **2017**, 411–418.

Heath, T. L. and Heiberg, J. L. (1908). *The thirteen books of Euclid's Elements Books 3-9 (Vol. 2)*. The University Press, pp. 24, 32, 88.

Hynes, M., Portsmore, M., Dare, E., Milto, E., Rogers, C., and Hammer, D. (2011). *Infusing engineering design into high school STEM courses.* Retrieved from the National Center for Engineering and Technology Education. The University Press. URL: http://ncete.org/flash/pdfs/Infusing%20Engineering%20Hynes.pdf.

Kilgore, D., Sattler, B., and Turns, J. (2013). "From fragmentation to continuity: Engineering students making sense of experience through the development of a professional portfolio". *Studies in Higher Education* **38**(6), 807–826. DOI: 10.1080/03075079.2011.610501.

Li, Y. (2014). "A platform to promote STEM education and research worldwide". *International Journal of STEM Education* **1**(1). DOI: 10.1186/2196-7822-1-1.

Li, Y. (2018). "Promoting the Development of Interdisciplinary Research in STEM Education". *Journal for STEM Education Research* **1**(6), 1–6. DOI: 10.1007/s41979-018-0009-z.

Li, Y. and Schoenfeld, A. (2019). "Problematizing teaching and learning mathematics as "given" in STEM education". *International Journal of STEM Education* **6**, 44. DOI: 10.1186/s40594-019-0197-9.

Loria, G. (1935). *Metodi matematici-Essenza-Tecniche-Applicazioni*. Hoepli, Milano.

Maass, K., Geiger, V., Romero Ariza, M., and Goos, M. (2019). "The Role of Mathematics in interdisciplinary STEM education". *ZDM* **51**, 1–16. DOI: 10.1007/s11858-019-01100-5.

McCallum, W. (2018). *Sense-making and making sense*. Retrieved on October 1, 2019. URL: https://blogs.ams.org/matheducation/2018/12/05/sense-making-and-making-sense/.

Monge, G. (1798). *Géométrie Descriptive. Leçons données aux écoles normales l'an III de la République*.

Recht, R. (2001). *Il disegno d'architettura: origini e funzioni*. Milano: Jaka Book.

Schoenfeld, A. (1992). "Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics". In: *NCTM Handbook of research on mathematics teaching and learning*. Ed. by D. Grouws. Macmillan, pp. 334–370.

Serpe, A. (2018). "Geometry of design in high school - an example of teaching with GeoGebra". In: *INTED2018, Proceedings 12th International Technology, Education and Development Conference*. Ed. by L. Gómez Chova, A. López Martinez, and I. Candel Torres. Valencia, Spain, pp. 463–477. DOI: 10.21125/inted.2018.0668.

Serpe, A. (2020). "A Computational Approach with MATLAB Software for Nonlinear Equation Roots Finding in High School Maths". In: *Numeri-cal Computations: Theory and Algorithms. NUMTA 2019. Lecture Notes in Computer Science, vol 11973*. Ed. by Y. Sergeyev and D. Kvasov. Springer, Cham, pp. 463–477. DOI: 10.1007/978-3-030-39081-5_40.

Serpe, A. and Frassia, M. G. (2018). “Legacy and Influence in Mathematics and Physics with Educational Technology: A Laboratory Example”. In: *New Trends in Physics Education Research*. Ed. by S. Magazù. Nova Science Publishers, pp. 77–96.

Serpe, A. and Frassia, M. G. (2020). “Task Mathematical Modelling Design in a Dynamic Geometry Environment: Archimedean Spiral’s Algorithm”. In: *Numerical Computations: Theory and Algorithms. NUMTA 2019. Lecture Notes in Computer Science, vol. 11973*. Ed. by S. Y. and K. D. Springer, Cham, pp. 478–491. DOI: 10.1007/978-3-030-39081-5_41.

Smith, J. and Karr - Kidwell, P. J. (2000). “The Interdisciplinary Curriculum: A Literary Review and a Manual for Administrators and Teachers”. In: ERIC.

Stillman, G. A., Blum, W., and Biembengut, M. S. (2015). “Cultural, social, cognitive and research influences on mathematical modelling education.” In: *Mathematical Modelling in Education Research and Practice*. Ed. by S. Y. and K. D. Springer, Cham, pp. 1–32. DOI: 10.1007/978-3-319-18272-8_1.

Tsupros, N., Kohler, R., and Hallinen, J. (2009). *TEM education: A project to identify the missing components. Intermediate Unit 1: Center for STEM Education and Leonard Gelfand Center for Service Learning and Outreach*. Carnegie Mellon University, Pennsylvania.

Vitruvius (1999). *Vitruvius: ’Ten Books on Architecture’*. Ed. by I. D. Rowland and T. N. Howe. translated by Ingrid D. Rowland, with commentary by others. Cambridge University Press. DOI: 10.1017/CBO9780511840951.

Voogt, J. and Pareja Roblin, N. (2012). “A comparative analysis of international frameworks for 21 st century competences: Implications for national curriculum policies”. *Journal of Curriculum Studies* **44**(3), 299–321. DOI: 10.1080/00220272.2012.668938.

Wang, H.-H., Moore, T. J., Roehrig, G. H., and Park, M. S. (2011). “STEM Integration: Teacher Perceptions and Practice”. *Journal of Pre-College Engineering Education Research (J-PEER)* **1**(2). DOI: 10.5703/1288284314636.

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