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PHASE DIAGRAM FOR ROEGENIAN ECONOMICS

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ABSTRACT. Firstly, this paper recall that the Roegenian Economics is created from a Thermodynamics-Economics dialog, transferring information via a specific dictionary. We have structured the explanations in an adapted dictionary, thermodynamics differential laws, economics differential laws, and integral manifolds of Gibbs-Pfaff economic equation. Secondly, we introduce and discuss the phase diagram for a Roegenian economic system highlighting a triple point and a critical point, with related explanations. These ideas can be used to improve knowledge and understanding of the nature of development and evolution of Roegenian economic systems.

1. Thermodynamics–Economics dictionary

Thermodynamics is important as a model of phenomenological theory, which describes and unifies certain properties of different types of physical systems. There are many systems in biology, economics and computer science, for which a similar and unitary-phenomenological organization would be desirable. Our purpose is to present certain features of the economy that are inspired by thermodynamics. In this context, we offer a Dictionary that reflects the Thermodynamics-Economics "isomorphism". The formal analytical-mathematical analogy between economics and thermodynamics is now well-known or at least accepted by economists and physicists as well. Econophysics denotes the activities of physicists who are working on economic problems to test a variety of new conceptual approaches deriving from the physical science and review the recent developments in the discipline and possible future trends (Chernavskiĭ *et al.* 2002; Ruth 2005; London and Tohme 2007; Sharma *et al.* 2011).

Recently, there have also been attempts of thermodynamics-economics dictionaries, but our dictionary below (see also Udrişte *et al.* 2002, 2004; Udrişte 2007; Udrişte *et al.* 2007, 2008, 2010, 2012, 2018a,b,c) is much more complete. In fact we build a "correspondence" between thermodynamics and economics, admitting that fundamental laws must be also in correspondence through our identification. Therefore, each thermodynamic system is

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naturally equivalent to an economic system, and thermodynamic laws have correspondence in economy.

In the following, we reproduce the correspondence between the characteristic state variables and the laws of thermodynamics with the macro-economics as described by Udrişte *et al.* (2002, 2004), Udrişte (2007), and Udrişte *et al.* (2007, 2008, 2010, 2012, 2018a,b,c), based on the theory on which the Roegenian economy was founded in 1971 (Georgescu-Roegen 1971). We do not know if Georgescu-Roegen considered this possibility and that is the reason why we have taken it into account instead him.

1.1. Basic thermodynamics differential laws. *The Gibbs-Pfaff fundamental equation in thermodynamics*, in the sense of Carathéodory, is

$$dU - TdS + PdV + \sum_k \mu_k dN_k = 0.$$

This equation is a combination of the first law and the second law (in thermodynamics). The third law of thermodynamics is $\lim_{T\to 0} S = 0$.

Process variables $W =$ *mechanical work* and $Q =$ *heat* are introduced into elementary mechanical thermodynamics by $dW = PdV$ (the first law) and by elementary heat, respectively, $dQ = TdS$, for reversible processes, or $dQ < TdS$, for irreversible processes (*second law*). The dimensional analysis shows that the Gibbs-Pfaff thermodynamic equation

$$dU - TdS + PdV = 0$$

is dimensionally correct. Indeed, we take into account the usual units of measure: U is the internal energy (SI unit: joule), P is pressure (SI unit: pascal), V is volume (SI unit: m^3), T is the temperature (SI unit: kelvin), S is the entropy (SI unit: joule per kelvin), and we check that all terms of the equation are equidimensional.

1.2. Adapted dictionary. An extended version of this dictionary can be found in the papers by Udrişte *et al.* (2004), Udrişte (2007), and Udrişte *et al.* (2007, 2008, 2018a,b).

THERMODYNAMICS	ECONOMICS
$U' =$ internal energy	$G' =$ growth potential
$T' =$ temperature	$I' =$ internal political stability
$S' =$ entropy	$E' =$ entropy
$P' =$ pressure	$P' =$ price level (inflation)
$V' =$ volume	$Q' =$ volume, structure, quality
$M' =$ total energy (mass)	$Y' =$ national income (income)
$\mu_k' =$ chemical potential	$\nu_k' =$ economical potential
$N_k' =$ number of moles	$\mathcal{N}_k' =$ number of economic moles
$W' =$ mechanical work	$W' =$ wealth of the system
$Q' =$ heat	$q' =$ stock market

From a mathematical point of view, here the thermodynamic and the economic variables are stripped of their true meaning and units of measure. However, for those interested in such a problem, we give a dimensional analysis for the Gibbs-Pfaff equation, in thermodynamics,

respectively in economics (one can also consult Neal and Shone 1976; Sonin 2001). As an example, a phase diagram (see Section 2) plots the price level (typically in reference currency) versus internal political stability (typically in percents - see, for instance, Moody's rating). The notion of economic mole ((for details, see Udrişte *et al.* 2018a,b) was introduced as a unit of measure of the value (merchandise value, money value).

1.3. Basic economics differential laws. Now we apply the previous dictionary, to obtain economics differential laws.

The Gibbs-Pfaff fundamental equation in thermodynamics is changed to *Gibbs-Pfaff fundamental equation of economics*

$$dG - IdE + PdQ + \sum_k \nu_k d\mathcal{N}_k = 0.$$

This last equation is a combination of the first law and the second law (in economy). The third law of economy $\lim_{I\to 0} E = 0$ has the following sense: "if that internal political stability I tends to 0, the system is blocked, meaning entropy becomes $E = 0$, equivalent to that fact that maintaining the functionality of the economic system must cause disruption".

The process variables in economics

$$W = \textit{wealth of system},\ q = \textit{stock market}$$

are defined by $dW = Pdq$ (*first economic law, elementary wealth in the economy*) and $dq = IdE$ or $dq < IdE$ (*second economic law, elementary production of commodities*). A *commodity* is an economic good, a product of human labor, with a utility in the sense of life, for sale-purchase on the market in an economy.

Remark 1.1. *The price level P related to commodities and the stock market q are independent variables in Roegenian economics, since their corespondents, the pressure P and heat Q, are independent in Carathéodory Thermodynamics.*

The usual measures used in an economic system are: GDP (Gross Domestic Product), for the wealth, and SI (stock index), for the Finance. The two measures (GDP and SI) reflect the two economy sides: the real economy related to the production of commodity and their trading, and the financial dynamics related to the stock market performances. The dimensional analysis shows that the Gibbs-Pfaff economic equation

$$dG - IdE + PdQ = 0$$

is dimensionally correct (satisfies the criterion of balance). Indeed, the usual economic units of measure are: G = growth potential (measure: monetary expenditure), I = internal political stability (measure: Moody's rating = percent), E = entropy (measure: monetary expenditure per Moody's rating), P = price level (measure: reference currency), Q = volume, structure, quality (measure: conventional units). All previous measures are specific and generally accepted into economy.

The long term association between Economics and Thermodynamics can be strengthened with new tools based on our dictionary (see also Udrişte *et al.* 2002, 2004; Udrişte 2007; Udrişte *et al.* 2007, 2008, 2010, 2012, 2018a,b,c). Of course, this new idea to build economics from thermodynamics, via a dictionary, produces new concepts in Econophysics (see also Chernavskiĭ *et al.* 2002; Ruth 2005; London and Tohme 2007; Sharma *et al.* 2011).

Our thermodynamics - economics dictionary allows the transfer of information from the first discipline to the second, keeping the background of each discipline, which we think that was suggested by Georgescu-Roegen (1971). A complementary point of view can be found in the papers by Ruth (2005) and London and Tohme (2007). Also the papers by Vollmer (2000) and Chernavskiĭ *et al.* (2002) complete our ideas.

To distinguish our creation from what we already know in Econophysics, we use the following

Definition 1.1. *The Economics which is structured similarly to thermodynamics, via previous dictionary, is called Roegenian economics.*

In the economic field, the ideas induced by the means of thermodynamics are preferable to mechanical ones. The motivation is that, the macroscopic thermodynamic quantities are not true point-wise mathematical functions. They are obtained through statistical mediation. Correspondingly, in economy the point-wise prediction is impossible (Isard 1954).

1.4. Integral manifolds of Gibbs – Pfaff economic equation. The Gibbs-Pfaff economic equation $\omega = dG - IdE + PdQ = 0$ has solutions which are either curves or surfaces, since ω is a contact form in the economic space $\mathbb{R}^5 = \{(G,I,E,P,Q)\}$.

We reproduce some details from Udrişte *et al.* (2004). The integral curves are images of C^1 regular mappings

$$\alpha\colon I \subset \mathbb{R} \to \mathbb{R}^5,\ \alpha(t) = (G(t),I(t),E(t),P(t),Q(t))$$

whose components verify the ODE $\dot{G} - I\dot{E} + P\dot{Q} = 0$. The integral surfaces are images of C^2 regular mappings

$$r\colon D \subset \mathbb{R}^2 \to \mathbb{R}^5,\ r(x,y) = (G(x,y),I(x,y),E(x,y),P(x,y),Q(x,y))$$

whose components verify the PDE system

$$\frac{\partial G}{\partial x} - I\frac{\partial E}{\partial x} + P\frac{\partial Q}{\partial x} = 0,\ \frac{\partial G}{\partial y} - I\frac{\partial E}{\partial y} + P\frac{\partial Q}{\partial y} = 0.$$

The integrability conditions

$$\frac{\partial^2 G}{\partial x \partial y} = \frac{\partial^2 G}{\partial y \partial x},\ \frac{\partial^2 E}{\partial x \partial y} = \frac{\partial^2 E}{\partial y \partial x},\ \frac{\partial^2 Q}{\partial x \partial y} = \frac{\partial^2 Q}{\partial y \partial x}$$

have as a consequence the equality of two Jacobians (second area condition, Maxwell equation)

$$\frac{\partial I}{\partial x}\frac{\partial E}{\partial y} - \frac{\partial E}{\partial x}\frac{\partial I}{\partial y} = \frac{\partial P}{\partial x}\frac{\partial Q}{\partial y} - \frac{\partial Q}{\partial x}\frac{\partial P}{\partial y}.$$

An integral surface of the Gibbs-Pfaff economic equation $\omega = 0$ is called a *simple economic system*. The parameters (variables) x, y are called *states* of the system. The nonholonomic hypersurface $(\mathbb{R}^5, \omega = 0)$ (designed either as a collection of integral curves or as a collection of integral surfaces, respectively) is called an *economic system*. From a local point of view the states x, y can be chosen as two of the five coordinates G, I, E, P, Q. In this sense we have 10 types of simple economic systems. The integral surfaces of the Gibbs-Pfaff economic equation $\omega = dG - IdE + PdQ = 0$ can be written as follows:

(i) using an arbitrary C^2 function $f(E,Q)$, when the integral surfaces are given by

$$G = f(E,Q),\ -I = \frac{\partial f}{\partial E},\ P = \frac{\partial f}{\partial Q};$$

(ii) using a C^2 generating function g; we write the equation in the form $dz + \langle x, dy\rangle = 0$; for any partition $A \cup B$ of the set $\{1,2\}$ in two sets and any C^2 function $g(x_A, y_B)$ of two variables in A and B, the formulas

$$y_A = \frac{\partial g}{\partial x_A},\ x_B = -\frac{\partial g}{\partial y_B},\ z = f - x_A \frac{\partial g}{\partial x_A} = f - x_A y_A$$

defines an integral surface.

In our case, $x_1 = -I, x_2 = P, y_1 = E, y_2 = Q, z = G$. There are 4 generating functions of the form $g = z + x_A y_A$: (1) $A = \varnothing$ implies $f_1 = G$; (2) $A = \{1\}$ implies $f_2 = G - EI$, free economic energy; (3) $A = \{2\}$ implies $f_3 = G + PQ$, wealth function; (4) $A = \{1,2\}$ implies $f_4 = G - EI + PQ$, economical potential.

Of course, the Cauchy problems associated to Gibbs-Pfaff economic equation do not have unique solutions. The fundamental Gibbs-Pfaff equation plays a central role in relating nonholonomic economics to statistical economics. This kind of problems will be studied in future papers.

2. Triple point in economic phase diagram

Let us think in the context of Roegenian economics. Often we find that different phases of a pure economic system can exist in equilibrium with one another. The phases on our diagram are just qualitative aspects. An economic system is a system of production, resource allocation and distribution of goods and services within a society or a given geographic area. It includes the combination of the various institutions, agencies, entities, decision-making processes and patterns of consumption that comprise the economic structure of a given community. As such, an economic system is a type of social system. The mode of production is a related concept. All economic systems have three basic questions to ask: what to produce, how to produce, in what quantities and who receives the output of production?

Figure 1 gives an example of a phase diagram that focuses the effect of the internal political stability and the level prices in an economic system, inspired by thermodynamics (Vollmer 2000), via the dictionary in Section 1, completed in this Section. According to this dictionary, the "pressure" is replaced by "price level" and the "temperature" is replaced by "internal political stability". Consequently, each point in the diagram represents a possible combination between the internal political stability and the level of prices for the studied economic phase. The $2D$ chart contains three regions that represent the correlation between different states of the economy: (1) inflation, (2) monetary policy of liquidity, (3) income. More precisely, we accept the following association chart:

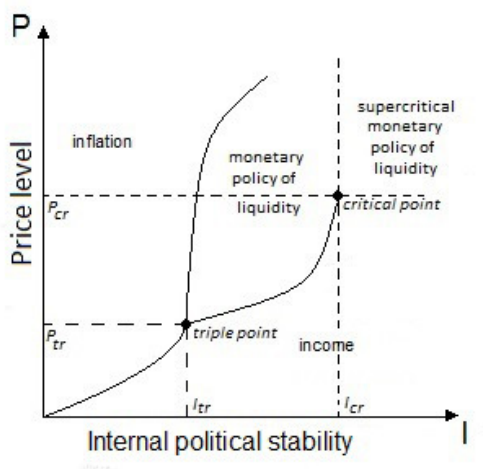

FIGURE 1. Triple point, critical point.

SUBSTANCE	ECONOMIC SYSTEM
$T =$ temperature (K)	$I =$ internal political stability (Mr = percent)
$P =$ pressure (Pa)	$P =$ price level (reference currency)
solid	inflation
fluid	monetary policy of liquidity
gas	income

The phase diagram shown in Fig. 1 is a graphical representation of the economic states of a Roegenian economical system under different conditions of internal political stability I and price level P. Our phase diagram has internal political stability I on the x-axis and price level P on the y-axis. As we cross the curves on the phase diagram, a phase change occurs. In addition, two states of the economic system coexist in equilibrium on some significant curves. The "apparently strange" association between "substance and economic system" (solid to inflation) is due to the signification of the horizontal axis (internal political stability). We recall some important phases in a Roegenian economical system.

Inflation. In economics, inflation is a sustained increase in the price level of goods and services in an economy over a period of time. When the price level rises, each unit of currency buys fewer goods and services; consequently, inflation reflects a reduction in the purchasing power per unit of money - a loss of real value in the medium of exchange and unit of account within the economy. A chief measure of price inflation is the inflation rate (percent).

Monetary policy of liquidity. Monetary policy is the process by which the monetary authority of a country, typically the central bank or currency board, controls either the cost of very short-term borrowing or the monetary base, often targeting an inflation rate or interest rate to ensure price stability and general trust in the currency. Liquidity means

how quickly you can get your hands on your cash. In simpler terms, liquidity is to get your money whenever you need it.

Consumption. Consumption is the using of goods and services in an economy, or the amount of goods and services used.

Income. In the field of public economics, the term income may refer to the accumulation of both monetary and non-monetary consumption ability, with the former (monetary) being used as a proxy for total income. Gross income can be defined as sum of all revenue. Net income = Revenue - Expenses.

In the following we assume that an economic system is found in one of the three previous states (inflation, monetary policy of liquidity, income). The stability in the domestic political and government actions are important contributions to attract the foreign investors to invest in the country. The foreign investment can make economic growth and developing host countries. The internal political un-stability and the high level of prices favor training inflation. Stability of increased domestic policy and low price levels generate income. Monetary policy as liquidity lies between these extremes.

Definition 2.1. *The common point of the three curves, "inflation - monetary policy of liquidity" equilibrium (recovery-recession curve), "monetary policy of liquidity - income" equilibrium (increasing-decreasing curve), "inflation - income" equilibrium (economic boom-crisis curve), is called triple point.*

Where all three curves meet, we must have a unique combination of internal politics stability and price level where all three phases are in equilibrium together. Generally, a phase diagram in physical chemistry, engineering, mineralogy, materials, and economics science is a type of 2D chart used to show that three states (c_1, c_2, c_3) coexist and they are at equilibrium depending on two primary indicators (p_1, p_2) on coordinate axes.

Related questions. Which are the implications of the triple point on real economic system issues? Do any economic system posse a triple point?

3. Critical point in economic phase diagram

Again, we relate to the similarity between a thermodynamic system (Vollmer 2000) and an economic system in the sense of the papers by Udrişte *et al.* (2002, 2004), Udrişte (2007), and Udrişte *et al.* (2007, 2008, 2010, 2012, 2018a,b,c), obtaining the 2D economic phase diagram in Fig. 1, in the context of Roegenian economics. The phase diagram illustrates the variations between the states of an economic system, "inflation, monetary policy of liquidity, and income", as they relate to internal political stability and price level. The previous phase diagram highlights two important points: a **triple point**, the point on a phase diagram at which the three states of an economic system, *i.e.*, inflation, monetary policy of liquidity, and income, coexist; a **critical point**, the point on a phase diagram at which the increasing-decreasing trend curve ends (see also Udrişte *et al.* 2018c).

Definition 3.1. *The point (I_c, P_c) is called critical point if the conditions $I > I_c$ (high internal political stability) and $P > P_c$ (high level prices) show that we attend supercritical monetary policy of liquidity (economic growth neutralizes high price levels, and conversely).*

The phase diagram plots the price level (typically in reference currency) versus internal political stability (typically in percents - see, for instance, Moody's rating). The labels on

the graph represent also the stable states of a system in equilibrium. The curves represent the combinations of the price level and internal political stability at which two phases can exist in equilibrium. In other words, these curves define phase change points. The curve between the inflation and income phases represents the economic boom (inflation to income) and crisis (income to inflation). The curve which joins the triple point and the critical point divides the income and monetary policy of liquidity phases and represents economic increasing (monetary policy of liquidity to income), or economic decreasing (income to monetary policy of liquidity). The curve between the inflation and the monetary policy of liquidity phases represents recovery-recession curve (inflation to monetary policy of liquidity - monetary policy of liquidity to inflation). There are also two important points on the diagram, the triple point and the critical point. The triple point represents the combination of price level and internal political stability that facilitates all the phases of an economic system at equilibrium. The critical point is the ending point of the "monetary policy of liquidity-income" phase curve and relates to the critical price level, the price level above, which a supercritical monetary policy of liquidity forms.

The critical point (I_{cr}, P_{cr}) is a projection of the degenerate critical point (I_{cr}, P_{cr}, Q_{cr}) of a constant level curve $I = const$ in the economic Van der Waals surface (for details, see Udrişte *et al.* 2018c). The Riemannian geometry of the economic Van der Waals surface will be studied in a future paper.

The triple point and the critical point reflect important changes in interior of economic $I - P$ diagram. In Roegenian economics, the triple point of an economic system is a pair (internal political stability, price level) at which the three phases (inflation, monetary policy of liquidity, and income) of that economic system coexist in economic equilibrium. Also, a critical point (or critical state) is the end point of a phase equilibrium curve in a Roegenian economic system.

4. Conclusions

Since the beginning of the nineteenth century, the laws of thermodynamics proved to be robust. We investigated deep connections between thermodynamics and economics theory (see Udrişte *et al.* 2002, 2004; Udrişte 2007; Udrişte *et al.* 2007, 2008, 2010, 2012, 2018a,b,c), connections that allow us to emphasize that Georgescu-Roegen's prediction is full of content (Georgescu-Roegen 1971). These links will lead to common explanations for the phenomena of Thermodynamics and Economics, sometimes new in Econophysics.

Roegenian economics (also called bioeconomics by Georgescu-Roegen) is both a transdisciplinary and an interdisciplinary field of academic research addressing the interdependence and coevolution of human economies and natural ecosystems, both intertemporally and spatially.

Moving about the phase diagram $I - P$ reveals information about the phases of an economic system. This interpretation holds in economics and comes from the thermodynamics - specific diagram. It was wonderful to find this parallelism between fields of Science (see also Udrişte *et al.* 2018a,b,c)). A key idea by which reveals a file rouge unifying different kind of human interactions with the nature. This is the great intuition of Georgescu-Roegen and for us was an incentive to develop this theory starting from a well-known scientific platform.

This paper aims to show how the corresponding notions of thermodynamics and Roegenian economics work. Obviously, the similarities cannot be total, but we propose a thermodynamic model of the economy, the Roegenian economics. In future works we shall give examples of economic systems characterized by the pairs (critical internal political stability, critical price levels).

High "internal political stability" phase diagrams including the critical point of income - "monetary policy of liquidity" phase transition as well as price-level (P,Q,I), production-level (q,Q,I) and entropic (E,Q,I) equations of states are very important in many aspects of the theoretical Roegenian economics.

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