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# TRANSITIVE COMBINATORIAL STRUCTURES INVARIANT UNDER SOME SUBGROUPS OF $S(6,2)$ AND RELATED CODES

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ABSTRACT. In this paper we define combinatorial structures on the conjugacy classes of the maximal subgroups of the symplectic group $S(6,2)$ under the action of two subgroups of $S(6,2)$ isomorphic to $U(3,3)$ and $U(4,2)$. Further, we examine binary and ternary linear codes obtained from the row span of the incidence matrices of the block designs (respectively adjacency matrices of the strongly regular graphs) obtained in the paper. Moreover, from the codes examined we construct the designs supported by the codewords as well as SRG and DRG, respectively.

## 1. Introduction

We assume that the reader is familiar with the basic facts of group theory, design theory and theory of strongly regular graphs. We refer the reader to the book of Tonchev (1988) for relevant background reading in design theory and to the book of Robinson (1989) for relevant background reading in group theory; for background reading in theory of strongly regular graphs see Tonchev (1988) and Brouwer (2007).

An incidence structure is an ordered triple $\mathscr{D} = (\mathscr{P}, \mathscr{B}, \mathscr{I})$ where $\mathscr{P}$ and $\mathscr{B}$ are nonempty disjoint sets and $\mathscr{I} \subseteq \mathscr{P} \times \mathscr{B}$. The elements of the set $\mathscr{P}$ are called points, the elements of the set $\mathscr{B}$ are called blocks and $\mathscr{I}$ is called an incidence relation. If $|\mathscr{P}| = |\mathscr{B}|$, then the incidence structure is called symmetric. The incidence matrix of an incidence structure is a $v \times b$ matrix $[m_{ij}]$ where $v$ and $b$ are the numbers of points and blocks respectively, such that $m_{ij} = 1$ if the point $P_i$ and the block $x_j$ are incident, and $m_{ij} = 0$ otherwise. An isomorphism from one incidence structure to another is a bijective mapping of points to points and blocks to blocks which preserves incidence. An isomorphism from an incidence structure $\mathscr{D}$ onto itself is called an automorphism of $\mathscr{D}$. The set of all automorphisms forms a group called the full automorphism group of $\mathscr{D}$ and is denoted by $\mathrm{Aut}(\mathscr{D})$.

A $t$-$(v,k,\lambda)$ design is a finite incidence structure $\mathscr{D} = (\mathscr{P}, \mathscr{B}, \mathscr{I})$ satisfying the following requirements:

(1) $|\mathscr{P}| = v$,
(2) every element of $\mathscr{B}$ is incident with exactly $k$ elements of $\mathscr{P}$,
(3) every $t$ elements of $\mathscr{P}$ are incident with exactly $\lambda$ elements of $\mathscr{B}$.

Note that this definition allows $\mathscr{B}$ to be a multiset. If $\mathscr{B}$ is a set then $\mathscr{D}$ is called a simple design. If the design $\mathscr{D}$ consists of $k$ copies of some simple design $\mathscr{D}'$ than $\mathscr{D}$ is non-simple design and it is denoted $\mathscr{D} = k\mathscr{D}'$. If $\mathscr{D}$ is a $t$-design, then it is also a $s$-design, for $1 \leq s \leq t-1$. A 2-$(v,k,\lambda)$ design is called a block design.

Let $\Gamma = (\mathscr{V}, \mathscr{E}, \mathscr{I})$ be a finite incidence structure. $\Gamma$ is a graph if each element of $\mathscr{E}$ is incident with exactly two elements of $\mathscr{V}$. The elements of $\mathscr{V}$ are called vertices and the elements of $\mathscr{E}$ are called edges. Two vertices $u$ and $v$ are called adjacent or neighbours if they are incident with the same edge. The number of neighbours of a vertex $v$ is called the degree of $v$. We define a square $\{0,1\}$-matrix $A = (a_{uv})$ labelled by the vertices of $\Gamma$ in such a way that $a_{uv} = 1$ if and only if the vertices $u$ and $v$ are adjacent. The matrix $A$ is called the adjacency matrix of the graph $\Gamma$.

A graph is regular if all the vertices have the same degree. A connected graph $\Gamma$ with diameter $d$ is called a distance-regular if there are integers $b_i, c_i, i \geq 0$ such that for any two vertices $u, v \in \Gamma$ at distance $d(u,v) = i$, there are exactly $c_i$ neighbours of $v$ at distance $i-1$ from $u$ and $b_i$ neighbours of $v$ at distance $i+1$ from $u$. The graph $\Gamma$ is a regular graph of valency $k = b_0$. The numbers $c_i, b_i, a_i$, where $a_i = k - b_i - c_i$, $i = 0, \ldots, d$ is the number of neighbours of $v$ at distance $i$ from $u$, are called the intersection numbers of $\Gamma$. The sequence $\{b_0, b_1, \ldots, b_{d-1}; c_1, c_2, \ldots, c_d\}$, where $d$ is the diameter of $\Gamma$ is called the intersection array of $\Gamma$. Clearly, $b_0 = k$, $b_d = c_0 = 0$, $c_1 = 0$.

A regular graph is strongly regular of type $(v,k,\lambda,\mu)$ if it has $v$ vertices, degree $k$, and if any two adjacent vertices are together adjacent to $\lambda$ vertices, while any two non-adjacent vertices are together adjacent to $\mu$ vertices. A strongly regular graph of type $(v,k,\lambda,\mu)$ is usually denoted by SRG$(v,k,\lambda,\mu)$. A strongly regular graph is a distance-regular graph with diameter 2 whenever $\mu \neq 0$. The intersection array of a strongly regular graph is given by $\{k, k-1-\lambda, 1, \mu\}$.

The code $C_{\mathbb{F}}$ of the design $\mathscr{D}$ over the finite field $\mathbb{F}$ is the space spanned by the incidence vectors of the blocks over $\mathbb{F}$. If $\mathscr{Q}$ is any subset of $\mathscr{P}$, then we will denote the incidence vector of $\mathscr{Q}$ by $v^{\mathscr{Q}}$. Thus $C_{\mathbb{F}} = \langle v^B \,|\, B \in \mathscr{B} \rangle$ is a subspace of $\mathbb{F}^{\mathscr{P}}$, the full vector space of functions from $\mathscr{P}$ to $\mathbb{F}$. Similarly, we can span a code by the incidence vectors of the points over some finite field $\mathbb{F}$. All our codes will be linear codes, i.e. subspaces of the ambient vector space. If a code $C$, over a field of order $q$, is of length $n$, dimension $k$, and minimum weight $d$, then we write $[n,k,d]_q$ to show this information. An automorphism of a code is any permutation of the coordinate positions that maps codewords to codewords. Two codes over a field of prime order are equivalent if one of the codes can be obtained from the other by permuting the coordinates and multiplication of components by non-zero elements.

The code of a graph $\Gamma$ over the finite field $\mathbb{F}$ is the row span of an adjacency matrix $A$ over the field $\mathbb{F}$.

The construction employed in this paper was introduced by Crnković *et al.* (2014). Crnković *et al.* (2016) defined incidence structures on the elements of the conjugacy classes of the maximal and second maximal subgroups of $S(6,2)$. In this paper we define combinatorial structures on the conjugacy classes of the maximal subgroups of the group $S(6,2)$ under the action of two subgroups of $S(6,2)$ isomorphic to $U(3,3)$ and $U(4,2)$. Moreover, we study the binary and ternary linear codes generated by the incidence matrices of the constructed block designs. The linear codes are spanned by incidence vectors of the points and the blocks. Additionally, we consider binary and ternary linear codes obtained from the adjacency matrices of the constructed strongly regular graphs. Further, the support designs and related SRGs and DRGs were constructed from obtained codes. Note that the linear codes that admit a primitive action of $S(6,2)$ are described by Chikamai (2012) and Chikamai *et al.* (2014), whereas the linear codes spanned by the incidence matrices of block designs and adjacency matrices of strongly regular graphs that admit a transitive action of $S(6,2)$ are described by Crnković *et al.* (2016).

All the structures are obtained with the help of Magma (Bosma *et al.* 1997) and *GAP* (2006). Generators of the group $S(6,2)$ are available on the Internet at http://brauer.maths.qmul.ac.uk/Atlas/.

The paper is organized as follows: in Section **2** we briefly describe the method of construction of transitive designs and graphs used in this paper, and in Sections **3** and **4**, we give a description of the interplay between the combinatorial structures obtained in the paper and the corresponding binary and ternary linear codes. In Section **5** we give details on the combinatorial designs constructed from the supports of the non-zero codewords in the related codes.

## 2. Construction method

We say that an incidence structure $\mathscr{I}$ is transitive if an automorphism group of $\mathscr{I}$ acts transitively on points and blocks. A transitive incidence structure $\mathscr{I}$ is called primitive if an automorphism group acts primitively on points and blocks. A flag of a design is an incident pair (point, block). We say that a $t$-design is flag-transitive if an automorphism group acts transitively on the set of flags of the design. Further, we say that a graph $\Gamma$ is transitive (primitive) if an automorphism group acts transitively (primitively) on the set of vertices of the graph $\Gamma$ and that a graph is edge-transitive if an automorphism group acts transitively on the set of edges of the graph.

The construction of primitive symmetric 1-designs and regular graphs for which the stabilizer of a point and the stabilizer of a block are conjugate is presented by Key and Moori (2002), Key *et al.* (2003), and Key and Moori (2008). The generalization, *i.e.* the method for constructing not necessarily symmetric but still primitive 1-designs, is presented by Crnković and Mikulić (2011, 2013). A construction of not necessarily primitive, but still transitive block designs is presented by Crnković *et al.* (2014).

**Theorem 2.1.** (Crnković *et al.* 2014) *Let $G$ be a finite permutation group acting transitively on the sets $\Omega_1$ and $\Omega_2$ of size $m$ and $n$, respectively. Let $\alpha \in \Omega_1$ and $\Delta_2 = \bigcup_{i=1}^{s} \delta_i G_\alpha$, where $G_\alpha = \{g \in G \mid \alpha g = \alpha\}$ is the stabilizer of $\alpha$ and $\delta_1, \dots, \delta_s \in \Omega_2$ are representatives of distinct $G_\alpha$-orbits on $\Omega_2$. If $\Delta_2 \neq \Omega_2$ and*

$$\mathscr{B} = \{\Delta_2 g : g \in G\},$$

*then* $\mathscr{D}(G,\alpha,\delta_1,\ldots,\delta_s) = (\Omega_2,\mathscr{B})$ *is a* $1$-$(n,|\Delta_2|,\frac{|G_\alpha|}{|G_{\Delta_2}|}\sum_{i=1}^{s}|\alpha G_{\delta_i}|)$ *design with* $\frac{m\cdot|G_\alpha|}{|G_{\Delta_2}|}$ *blocks. The group* $H \cong G/\bigcap_{x\in\Omega_2} G_x$ *acts as an automorphism group on* $(\Omega_2,\mathscr{B})$*, transitively on points and blocks of the design.*

*If* $\Delta_2 = \Omega_2$ *then the set* $\mathscr{B}$ *consists of one block, and* $\mathscr{D}(G,\alpha,\delta_1,\ldots,\delta_s)$ *is a design with parameters* $1$-$(n,n,1)$.

If a group $H$ acts $t$-homogeneously on the set $\Omega_2$, then the obtained design $(\Omega_2,\mathscr{B})$ is a $t$-design.

The construction described in Theorem **2.1** gives us all simple designs on which the group $G$ acts transitively on the points and blocks, *i.e.* if $G$ acts transitively on the points and blocks of a simple 1-design $\mathscr{D}$, then $\mathscr{D}$ can be obtained as described in Theorem **2.1**. It follows from Proposition 1.3. of Cameron and Praeger (1993) that the group $H$ acts flag-transitively on the design $(\Omega_2,\mathscr{B})$ if and only if the base block $\Delta_2$ is a single $G_\alpha$-orbit.

**Corollary 2.2.** *If* $\Omega_1 = \Omega_2$*,* $\Delta_2$ *is a union of self-paired and mutually paired orbits of* $G_\alpha$*, and* $G_\alpha = G_{\Delta_2}$*, then the design* $\mathscr{D}(G,\alpha,\delta_1,\ldots,\delta_s)$ *is a symmetric self-dual design and the incidence matrix of that design is the adjacency matrix of a* $|\Delta_2|$*-regular graph.*

**Proof**. The obtained incidence matrix is symmetric with zero diagonal. Hence, it is the incidence matrix of a symmetric self-dual 1-design, and also the adjacency matrix of a $|\Delta_2|$-regular graph. □

If $G_\alpha = G_{\Delta_2}$, we can interpret the design $(\Omega_2,\mathscr{B})$ from Theorem **2.1** in the following way:

- the point set is $\Omega_2$,
- the block set is $\Omega_1 = \alpha G$,
- the block $\alpha g'$ is incident with the set of points $\{\delta_i g : g \in G_\alpha g', i = 1,\ldots,s\} = \Delta_2 g'$.

If $G_\alpha \neq G_{\Delta_2}$, then the above described design with the block set $\Omega_1$ has repeated blocks, and the corresponding simple design is the design $(\Omega_2,\mathscr{B})$ from Theorem **2.1**, which has $\frac{m\cdot|G_\alpha|}{|G_{\Delta_2}|}$ blocks.

If a point $\delta_1 g$ is incident with the block $\alpha$ then $G_\alpha \cap G_{\delta_1 g} = (G_\alpha \cap G_{\delta_1})^g$, and if a point $\delta_1 g$ is incident with a block $\alpha g'$ then $G_{\alpha g'} \cap G_{\delta_1 g} = (G_\alpha \cap G_{\delta_1})^g$ (see Crnković and Mikulić 2013). Consequently, for each $G_\alpha$-orbit $\delta_i G_\alpha \subset \Omega_2$ there is the corresponding $G_\alpha$-conjugacy class on the set $\{G_\alpha \cap G_{\delta_1 g} \mid g \in G\}$. Note that this is not necessarily an injection. Further, for $G_\alpha$-conjugacy class on the set $\{G_\alpha \cap G_{\delta_1 g} \mid g \in G\}$ there is the corresponding isomorphism class on that set (again, this is not necessarily an injection).

**2.1. Construction from conjugacy classes.** Let $M$ be a finite group and $H_1$, $H_2$, and $G$ be subgroups of $M$. Then the group $M$ acts (by conjugation) transitively on the conjugacy classes $ccl_M(H_1)$ and $ccl_M(H_2)$. Further, the group $G$, being a subgroup of the group $M$, acts transitively on the class $ccl_G(H_i)$, $i = 1,2$, by conjugation. Let us set the following:

$$|ccl_G(H_1)| = [G : N_G(H_1)] = m,$$

$$|ccl_G(H_2)| = [G : N_G(H_2)] = n.$$

We can use Theorem **2.1** to construct 1-design as follows. Note that the resulting 1-design may have repeated blocks.

**Theorem 2.3.** *Let $M$ be a finite group and $H_1$, $H_2$, and $G$ be subgroups of $M$. Denote the elements of $ccl_G(H_1)$ by $H_1^{g_1}, H_1^{g_2}, \ldots, H_1^{g_m}$, and the elements of $ccl_G(H_2)$ by $H_2^{h_1}, H_2^{h_2}, \ldots, H_2^{h_n}$. Define an incidence structure as follows:*

- *The point set of the design is $ccl_G(H_2)$.*
- *The block set of the design is $ccl_G(H_1)$.*
- *Let $\{G_1, \ldots, G_p\}$ be a subset of $\{H_2^x \cap H_1^y \mid x, y \in G\}$. The block $H_1^{g_i}$ is incident with the point $H_2^{h_j}$ if and only if there exists $u \in \{1, \ldots, p\}$ such that $H_2^{h_j} \cap H_1^{g_i} \cong G_u$, i.e. The base block $H_1$ is incident with the elements of the set $\{H_2^x \mid x \in G, (\exists u \in \{1, \ldots, p\})$ s.t. $H_2^x \cap H_1 \cong G_u\}$.*

*Let $\Omega_2 = ccl_G(H_2)$ and $\mathscr{B} = \{\Delta_2^g : g \in G\}$. Then $\mathscr{D}(G, H_2, H_1; G_1, \ldots, G_p) = (\Omega_2, \mathscr{B})$ is a 1-design and the group $G$ acts transitively on the points and the blocks of $\mathscr{D}$.*

**Proof**. The group $G$ acts on the sets $ccl_G(H_1)$ and $ccl_G(H_2)$ by conjugation. Let us set the following:

- $\Omega_1 = ccl_G(H_1)$,
- $\alpha = H_1$,
- $\Delta_2 = \{X \in ccl_G(H_2) \mid (\exists u \in \{1, \ldots, p\}) \text{ s.t. } X \cap H_1 \cong G_u\}$.

It is clear that $G_\alpha = N_M(H_1) \cap G$ and $\Delta_2$ is a union of $G_\alpha$-orbits on the set $ccl_G(H_2)$. Applying Theorem **2.1** concludes the proof. □

As stated before, the group $G$ acts by conjugation on the conjugacy classes $ccl_G(H_1)$ and $ccl_G(H_2)$. The stabilizer of the group $H_2$ for the given transitive action of the group $G$ on the conjugacy class $ccl_G(H_2)$ is $S_2 = N_M(H_2) \cap G$. For each $S_2$-orbit on the class $ccl_G(H_2)$ there is the corresponding isomorphism class on the set $\{N_M(H_2^g) \cap N_M(H_1) \cap G \mid g \in G\}$, and consequently, the corresponding isomorphism class on the set $\{H_2^g \cap H_1 \mid g \in G\}$, which is not necessarily an injection.

The necessary condition for flag-transitive action of the group $G$ on $\mathscr{D}(G, H_2, H_1; G_1, \ldots, G_p)$ is that $p = 1$, *i.e.* the incidence relation is defined by the group $G_1$ only. This condition is not sufficient. The group $G$ acts flag-transitively on a design $\mathscr{D}(G, H_2, H_1; G_1)$ if and only if the base block (*i.e.* the elements $H_2^{x_1}, \ldots, H_2^{x_k}$ such that the set $\{H_2^{x_1} \cap H_1, \ldots, H_2^{x_k} \cap H_1\}$ is the isomorphism class on the set $\{H_2^g \cap H_1 \mid g \in G\}$ represented with the group $G_1$) is $S_2$-orbit on the class $ccl_G(H_2)$.

**Corollary 2.4.** *Let $M$ be a finite group and $H$ and $G$ be subgroups of $M$. Let $\Gamma(G, H; G_1, \ldots, G_p)$ denote the graph constructed as follows:*

- *The vertex set of the graph is $ccl_G(H)$.*
- *Let $\{G_1, \ldots, G_p\}$ be a subset of $\{H^x \cap H^y \mid x, y \in G\}$. The vertex $H^{g_i}$ is adjacent to the vertex $H^{g_j}$ if and only if there exists $u \in \{1, \ldots, p\}$ such that $H^{h_j} \cap H^{g_i} \cong G_u$.*

*Then, $\Gamma(G, H; G_1, \ldots, G_p)$ is regular and $G$ acts as an automorphism group of $\Gamma(G, H; G_1, \ldots, G_p)$.*

**Proof**. This is a direct consequence of Theorem **2.3** and Corollary **2.2**. □

The adjacency matrix is the incidence matrix of the symmetric design $\mathscr{D}(G,H;G_1,\ldots,G_p)$. Hence, the group $G$ acts edge-transitively on the constructed regular graph if and only if it acts flag-transitively on the corresponding 1-design.

**Remark 2.5.** *Note that we only consider* 1*-designs that are* 2*-designs and regular graphs that are strongly regular.*

## 3. Combinatorial structures constructed from $S(6,2)$

The group $S(6,2)$ has 1993 maximal subgroups, and has 8 distinct $S(6,2)$-conjugacy classes of the maximal subgroups $M_1, M_2, \ldots, M_8$, given in Table 1.

| Subgroup | Structure of the subgroup | Order | Size of $G$-conjugacy class |
|---|---|---|---|
| $M_8$ | $U(4,2):Z_2$ | 51840 | 28 |
| $M_7$ | $S_8$ | 40320 | 36 |
| $M_6$ | $E_{32}:S_6$ | 23040 | 63 |
| $M_5$ | $U(3,3):Z_2$ | 12096 | 120 |
| $M_4$ | $E_{64}:L(3,2)$ | 10752 | 135 |
| $M_3$ | $((E_{16}:Z_2)\times E_4):(S_3\times S_3)$ | 4608 | 315 |
| $M_2$ | $S_3\times S_6$ | 4320 | 336 |
| $M_1$ | $L(2,8):Z_3$ | 1512 | 960 |

TABLE 1. Maximal subgroups of the group $S(6,2)$ up to $S(6,2)$-conjugation.

We define structures on the conjugacy classes of the maximal subgroups of the group $S(6,2)$ under the action of two subgroups $U(3,3)$ and $U(4,2)$. We do not need to consider conjugacy classes of all maximal subgroups, we can eliminate some of them. We search for all those maximal subgroups of the $S(6,2)$ which are not conjugate under the action of the groups $U(3,3)$ and $U(4,2)$, respectively. Finally, after elimination, we got 14 maximal subgroups of the group $S(6,2)$, which are not conjugate under the action of the subgroup $U(3,3)$ and 12 maximal subgroups of the group $S(6,2)$ which are not conjugate under the action of the subgroup $U(4,2)$. Information about the subgroups are given in Tables 2 and 3.

| Group | Structure of the group | Order | Size of the class |
|---|---|---|---|
| $N_1^1$ | $U(4,2){:}Z_2$ | 51840 | 28 |
| $N_2^1$ | $S_8$ | 40320 | 36 |
| $N_3^1$ | $E_{32}{:}S_6$ | 23040 | 63 |
| $N_4^1$ | $U(3,3){:}Z_2$ | 12096 | 1 |
| $N_5^1$ | $U(3,3){:}Z_2$ | 12096 | 63 |
| $N_6^1$ | $U(3,3){:}Z_2$ | 12096 | 56 |
| $N_7^1$ | $E_{64}{:}L(3,2)$ | 10752 | 36 |
| $N_8^1$ | $E_{64}{:}L(3,2)$ | 10752 | 36 |
| $N_9^1$ | $E_{64}{:}L(3,2)$ | 10752 | 63 |
| $N_{10}^1$ | $((E_{16}{:}Z_2)\times E_4){:}(S_3\times S_3)$ | 4608 | 63 |
| $N_{11}^1$ | $((E_{16}{:}Z_2)\times E_4){:}(S_3\times S_3)$ | 4608 | 252 |
| $N_{12}^1$ | $S_3\times S_6$ | 4320 | 336 |
| $N_{13}^1$ | $L(2,8){:}Z_3$ | 1512 | 288 |
| $N_{14}^1$ | $L(2,8){:}Z_3$ | 1512 | 672 |

TABLE 2. Maximal subgroups of the group $S(6,2)$ up to $U(3,3)$-conjugation.

| Group | Structure of the group | Order | Size of the class |
|---|---|---|---|
| $N_1^2$ | $U(4,2){:}Z_2$ | 51840 | 27 |
| $N_2^2$ | $U(4,2){:}Z_2$ | 51840 | 1 |
| $N_3^2$ | $S_8$ | 40320 | 36 |
| $N_4^2$ | $E_{32}{:}S_6$ | 23040 | 36 |
| $N_5^2$ | $E_{32}{:}S_6$ | 23040 | 27 |
| $N_6^2$ | $U(3,3){:}Z_2$ | 12096 | 120 |
| $N_7^2$ | $E_{64}{:}L(3,2)$ | 10752 | 135 |
| $N_8^2$ | $((E_{16}{:}Z_2)\times E_4){:}(S_3\times S_3)$ | 4608 | 270 |
| $N_9^2$ | $((E_{16}{:}Z_2)\times E_4){:}(S_3\times S_3)$ | 4608 | 45 |
| $N_{10}^2$ | $S_3\times S_6$ | 4320 | 216 |
| $N_{11}^2$ | $S_3\times S_6$ | 4320 | 120 |
| $N_{12}^2$ | $L(2,8){:}Z_3$ | 1512 | 960 |

TABLE 3. Maximal subgroups of the group $S(6,2)$ up to $U(4,2)$-conjugation.

In Table 4 we give block designs constructed on the conjugacy classes of the maximal subgroups of the group $S(6,2)$ under the action of the group $U(3,3)$.

| Block design $\mathscr{D}$ | Parameters of $\mathscr{D}$ | Simple design | Aut($\mathscr{D}$) |
|---|---|---|---|
| $\mathscr{D}_1 = \mathscr{D}(U(3,3), N_1^1, N_3^1, (E_{16}:A_5):Z_2)$ | $(28,12,11)^*$ | yes | $S(6,2)$ |
| $\mathscr{D}_2 = \mathscr{D}(U(3,3), N_1^1, N_{10}^1, (SL(2,3):E_4):Z_{12})$ | $(28,4,1)^*$ | yes | $U(3,3):Z_2$ |
| $\mathscr{D}_3 = \mathscr{D}(U(3,3), N_1^1, N_{11}^1, (SL(2,3):E_4):Z_{12})$ | $(28,4,4)^*$ | yes | $U(3,3):Z_2$ |
| $\mathscr{D}_4 = \mathscr{D}(U(3,3), N_1^1, N_{12}^1, ((S_3 \times S_3):Z_2) \times S_3)$ | $(28,10,40)$ | yes | $S(6,2)$ |
| $\mathscr{D}_5 = \mathscr{D}(U(3,3), N_2^1, N_3^1, Z_2 \times S_6)$ | $(36,16,12)^*$ | yes | $S(6,2)$ |
| $\mathscr{D}_6 = \mathscr{D}(U(3,3), N_2^1, N_{12}^1, S_5 \times S_3)$ | $(36,6,8)^*$ | yes | $S(6,2)$ |
| $\mathscr{D}_7 = \mathscr{D}(U(3,3), N_3^1, N_3^1, (E_{32}:A_6):Z_2,$ $E_4 \times (((Z_2 \times D_8):Z_2):Z_6))$ | $(63,31,15)$ | yes | $PGL(6,2)$ |
| $\mathscr{D}_8 = \mathscr{D}(U(3,3), N_7^1, N_7^1, L(3,2), E_{64}:L(3,2))$ | $(36,15,6)$ | yes | $U(3,3):Z_2$ |
| $\mathscr{D}_9 = \mathscr{D}(U(3,3), N_{10}^1, N_{10}^1, E_4 \times D_8,$ $(E_4 \times (E_{16}:Z_2)):Z_2, (E_4 \times (Z_2 \times D_8):Z_6):Z_{12})$ | $(63,31,15)$ | yes | $U(3,3):Z_2$ |

TABLE 4. Block designs constructed from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(3,3)$.

In Table 5 we give block designs constructed on the conjugacy classes of the maximal subgroups of the group $S(6,2)$ under the action of the group $U(4,2)$.

| Block design $\mathscr{D}$ | Parameters of $\mathscr{D}$ | Simple design | Corresponding simple design | Aut($\mathscr{D}$) |
|---|---|---|---|---|
| $\widetilde{\mathscr{D}_1} = \mathscr{D}(U(4,2), N_3^2, N_7^2, E_8 \times L(3,2))$ | $(36,8,6)$ | yes | | $S(6,2)$ |
| $\widetilde{\mathscr{D}_2} = \mathscr{D}(U(4,2), N_4^2, N_4^2, E_4 \times ((Z_2 \times D_8):Z_6):Z_2)$ | $(36,15,6)^*$ | yes | | $U(4,2):Z_2$ |
| $\widetilde{\mathscr{D}_3} = \mathscr{D}(U(4,2), N_9^2, N_7^2, (E_8 \times D_8):Z_2)$ | $(45,12,9)$ | no | $(45,12,3)$ | $U(4,2):Z_2$ |
| $\widetilde{\mathscr{D}_4} = \mathscr{D}(U(4,2), N_9^2, N_9^2, E_8 \times S_4)$ | $(45,12,3)^*$ | yes | | $U(4,2):Z_2$ |
| $\widetilde{\mathscr{D}_5} = \mathscr{D}(U(4,2), N_9^2, N_{11}^2, S_3 \times S_3)$ | $(45,12,8)^*$ | yes | | $U(4,2):Z_2$ |

TABLE 5. Block designs constructed from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(4,2)$.

In Tables 6 and 7 we give a list of strongly regular graphs constructed on the conjugacy classes of the maximal subgroups of the group $S(6,2)$ under the action of the groups $U(3,3)$ or $U(4,2)$, respectively.

| Graph $\Gamma$ | Parameters of $\Gamma$ | Aut($\Gamma$) |
|---|---|---|
| $\Gamma_1 = \Gamma(U(3,3), N_3^1, E_4 \times (((Z_2 \times D_8) : Z_2) : Z_6))$ | $(63,30,13,15)$ | $S(6,2)$ |
| $\Gamma_2 = \Gamma(U(3,3), N_{10}^1, E_4 \times D_8, (E_4 \times (E_{16} : Z_2)) : Z_2)$ | $(63,30,13,15)$ | $U(3,3) : Z_2$ |
| $\Gamma_3 = \Gamma(U(3,3), N_7^1, L(3,2))$ | $(36,14,4,6)$ | $U(3,3) : Z_2$ |

TABLE 6. SRGs constructed from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(3,3)$.

| Graph $\Gamma$ | Parameters of $\Gamma$ | Aut($\Gamma$) |
|---|---|---|
| $\widetilde{\Gamma_1} = \Gamma(U(4,2), N_4^2, E_4 \times (((Z_2 \times D_8) : Z_2) : Z_6))$ | $(36,15,6,6)^*$ | $U(4,2) : Z_2$ |
| $\widetilde{\Gamma_2} = \Gamma(U(4,2), N_5^2, E_4 \times (((Z_2 \times D_8) : Z_2) : Z_6))$ | $(27,10,1,5)^*$ | $U(4,2) : Z_2$ |
| $\widetilde{\Gamma_3} = \Gamma(U(4,2), N_6^2, (E_9 : Z_3) : Q_8)$ | $(120,56,28,24)$ | $O_8^+(2) : Z_2$ |
| $\widetilde{\Gamma_4} = \Gamma(U(4,2), N_6^2, L(3,2))$ | $(135,64,28,32)$ | $O_8^+(2) : Z_2$ |
| $\widetilde{\Gamma_5} = \Gamma(U(4,2), N_9^2, E_8 \times S_4)$ | $(45,12,3,3)^*$ | $U(4,2) : Z_2$ |

TABLE 7. SRGs constructed from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(4,2)$.

**Remark 3.1.** *The designs marked with $*$ in Tables 4 and 5 are flag-transitive, with respect to the groups $U(3,3)$ and $U(4,2)$, respectively. The designs $\mathscr{D}_7$, $\mathscr{D}_8$ and $\mathscr{D}_9$ do not meet the necessary condition for the flag-transitivity given in Section **2.1** whereas the designs $\mathscr{D}_4$ and $\widetilde{\mathscr{D}_1}$ meet the necessary but not the sufficient condition.*

*Additionally, we obtained more flag-transitive designs that are not mentioned in Tables 4 and 5, since they are the complements of the mentioned designs. These flag-transitive designs have the parameters $(28,16,20)$, $(28,24,46)$, $(28,18,136)$, $(36,21,12)$, $(63,32,16)$.*

*The strongly regular graphs with $*$ in Tables 6 and 7 are edge-transitive. Additionally, we obtained more edge-transitive graphs that are not mentioned in Tables 6 and 7, since they are the complements of the mentioned graphs. These are the complements of the SRGs $\Gamma_2$, $\Gamma_3$, $\widetilde{\Gamma_1}$, $\widetilde{\Gamma_2}$ and $\widetilde{\Gamma_5}$.*

*More details about the listed designs and graphs can be found in the article of Crnković* et al. *(2016).*

## 4. Codes

We describe codes of the constructed simple designs and their complements. If $A$ is an incidence matrix of a 2-$(v,k,\lambda)$ design $\mathscr{D}$ and $p$ is a prime that does not divide $r-\lambda$, then $\mathrm{rank}_p(A) \geq v-1$ (see Tonchev 2007). If $\mathrm{rank}_p(A) < v-1$ then $p$ divides $r-\lambda$, hence the code of a design $\mathscr{D}$ is interesting only when $p$ divides $r-\lambda$.

In Tables 8, 9, 10 and 11 we present the non-trivial codes of the constructed simple block designs and their complements. Further, in Tables 12 and 13 we present information about the non-trivial codes obtained from the strongly regular graphs constructed in this paper. In tables to follow, codes indicated by $*$ are optimal.

| Design | Parameters | $\lvert\mathrm{Aut}(C)\rvert$ | $\mathrm{Aut}(C)$ | Self-orthogonal |
|---|---|---|---|---|
| $\mathscr{D}_1$ | $[28,7,12]_2$ $*$ | 1451520 | $S(6,2)$ | yes |
| $\mathscr{D}_2$ | $[28,21,4]_2$ $*$ | 1451520 | $S(6,2)$ | no |
| $\mathscr{D}_5$ | $[36,7,16]_2$ $*$ | 1451520 | $S(6,2)$ | yes |
| $\mathscr{D}_6$ | $[36,21,6]_2$ | 1451520 | $S(6,2)$ | no |
| $\mathscr{D}_7$ | $[63,7,31]_2$ $*$ | 20158709760 | $PGL(6,2)$ | no |
| $\mathscr{D}_8$ | $[36,14,12]_3$ | 2903040 | $O(7,2)\times Z_2$ | yes |
| $\mathscr{D}_9$ | $[63,15,6]_2$ | 12096 | $U(3,3):Z_2$ | no |

TABLE 8. Non-trivial codes spanned by the blocks of the incidence matrices of the designs (from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(3,3)$).

| Design | Parameters | $\lvert\mathrm{Aut}(C)\rvert$ | $\mathrm{Aut}(C)$ | Self-orthogonal |
|---|---|---|---|---|
| $\mathscr{D}_1$ | $[63,7,27]_2$ | 1451520 | $S(6,2)$ | no |
| $\mathscr{D}_2$ | $[63,21,9]_2$ | 12096 | $U(3,3):Z_2$ | no |
| $\mathscr{D}_3$ | $[252,21,36]_2$ | 12096 | $U(3,3):Z_2$ | yes |
| $\mathscr{D}_4$ | $[336,21,96]_2$ | 1451520 | $S(6,2)$ | yes |
| $\mathscr{D}_4$ | $[336,27]_5$ | 1451520 | $S(6,2)$ | yes |
| $\mathscr{D}_5$ | $[63,7,28]_2$ | 1451520 | $S(6,2)$ | yes |
| $\mathscr{D}_6$ | $[336,21,56]_2$ | 1451520 | $S(6,2)$ | yes |
| $\mathscr{D}_6$ | $[336,35,56]_3$ | 2903040 | $O(7,2)\times Z_2$ | no |

TABLE 9. Non-trivial codes spanned by the points of the incidence matrices of the designs (from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(3,3)$).

| Design | Parameters | $\|Aut(C)\|$ | $Aut(C)$ | Self-orthogonal |
|---|---|---|---|---|
| $\widetilde{\mathscr{D}_1}$ | $[36,15,8]_2$ | 1451520 | $S(6,2)$ | no |
| $\widetilde{\mathscr{D}_5}$ | $[45,14,12]_2$ | 51840 | $U(4,2):Z_2$ | yes |

TABLE 10. Non-trivial codes spanned by the blocks of the incidence matrices of the designs (from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(4,2)$).

| Design | Parameters | $\|Aut(C)\|$ | $Aut(C)$ | Self-orthogonal |
|---|---|---|---|---|
| $\widetilde{\mathscr{D}_1}$ | $[135,15,30]_2$ | 1451520 | $S(6,2)$ | yes |
| $\widetilde{\mathscr{D}_1}$ | $[135,36,15]_3$ | 2903040 | $O(7,2)\times Z_2$ | yes |
| $\widetilde{\mathscr{D}_2}$ | $[36,15,9]_3$ | 103680 | $(U(4,2):Z_2)\times Z_2$ | yes |
| $\widetilde{\mathscr{D}_4}$ | $[45,15,12]_3$ | 103680 | $(U(4,2):Z_2)\times Z_2$ | yes |
| $\widetilde{\mathscr{D}_5}$ | $[120,14,32]_2$ | 51840 | $U(4,2):Z_2$ | yes |
| $\widetilde{\mathscr{D}_5}$ | $[120,44,18]_3$ | 103680 | $(U(4,2):Z_2)\times Z_2$ | no |

TABLE 11. Non-trivial codes spanned by the points of the incidence matrices of the designs (from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(4,2)$).

| Graph | Parameters | $\|Aut(C)\|$ | $Aut(C)$ | Self-orthogonal |
|---|---|---|---|---|
| $\Gamma_2$ | $[36,8,14]_2$ | 12096 | $U(3,3):Z_2$ | yes |
| $\Gamma_3$ | $[63,27,12]_3$ | 24192 | $(U(3,3):Z_2)\times Z_2$ | no |

TABLE 12. Non-trivial codes spanned by the rows of the adjacency matrices of the graphs (from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(3,3)$).

| Graph | Parameters | $\|Aut(C)\|$ | $Aut(C)$ | Self-orthogonal |
|---|---|---|---|---|
| $\widetilde{\Gamma_3}$ | $[120,8,56]_2$ | 348364800 | $O_8^+(2):Z_2$ | yes |

TABLE 13. Non-trivial codes spanned by the rows of the adjacency matrices of the graphs (from the group $S(6,2)$, from the conjugacy classes of maximal subgroups under the action of the subgroup $U(4,2)$).

## 5. Combinatorial structures from codes

The support of a nonzero vector $x = (x_1,\dots,x_n) \in F_q^n$ is the set of indices of its nonzero coordinates, i.e. $\mathrm{supp}(x) = \{i | x_i \neq 0\}$. The support design of a code of length $n$ for a given nonzero weight $w$ is the design with points the $n$ coordinate indices and blocks the supports of all codewords of weight $w$. Let $S = \{|x \cap y|\ x, y \in \mathscr{B}\}$, where $\mathscr{B}$ is the block set of the support design, and let $A \subset S$. One can define a graph whose vertices are the elements of the set $\mathscr{B}$ and two vertices are adjacent if the size of the intersection of the corresponding blocks is an element of $A$.

In Tables 14, 15 and 16 we list $t$-designs, strongly regular graphs and distance-regular graphs constructed from support designs of codes described in Section **4**.

| $\mathscr{C}$ | $\mathscr{D}$ | Aut($\mathscr{D}$) |
|---|---|---|
| $[28,7,12]_2$ | 2-$(28,12,11), b = 63$ | $S(6,2)$ |
| $[28,21,4]_2$ | 2-$(28,4,5), b = 315$ | $S(6,2)$ |
| | 2-$(28,6,240), b = 6048$ ∗ | $S(6,2)$ |
| | 2-$(28,8,3542), b = 47817$ ∗ | $S(6,2)$ |
| | 2-$(28,10,24640), b = 206976$ ∗ | $S(6,2)$ |
| | 2-$(28,12,82423), b = 472059$ ∗ | $S(6,2)$ |
| | 3-$(28,14,70080), b = 630720$ ∗ | $S(6,2)$ |
| $[36,7,16]_2$ | 2-$(36,16,12), b = 63$ | $S(6,2)$ |
| $[36,21,6]_2$ | 2-$(36,6,8), b = 336$ | $S(6,2)$ |
| | 2-$(36,8,42), b = 945$ | $S(6,2)$ |
| | 2-$(36,10,1152), b = 16128$ ∗ | $S(6,2)$ |
| | 2-$(36,12,8217), b = 78435$ ∗ | $S(6,2)$ |
| | 2-$(36,14,33176), b = 229680$ ∗ | $S(6,2)$ |
| | 2-$(36,16,83964), b = 440811$ ∗ | $S(6,2)$ |
| | 3-$(36,18,64512), b = 564480$ ∗ | $S(6,2)$ |
| $[63,7,31]_2$ | 2-$(63,31,15), b = 63$ | $PGL(6,2)$ |
| $[36,14,12]_3$ | 2-$(36,12,132), b = 1260$ ∗ | $S(6,2)$ |
| | 2-$(36,15,720), b = 4320$ ∗ | $S(6,2)$ |
| | 2-$(36,18,28866), b = 118860$ ∗ | $S(6,2)$ |
| | 2-$(36,15,93792), b = 562752$ ∗ | $S(6,2)$ |
| | 2-$(36,12,102663), b = 979965$ ∗ | $S(6,2)$ |
| | 2-$(36,9,35488), b = 621040$ ∗ | $S(6,2)$ |
| | 2-$(36,6,1628), b = 68376$ ∗ | $S(6,2)$ |
| | 2-$(36,3,18), b = 3780$ | $S(6,2)$ |
| $[36,15,8]_2$ | 2-$(36,8,6), b = 135$ | $S(6,2)$ |
| | 2-$(36,12,99), b = 945$ | $S(6,2)$ |
| | 2-$(36,14,624), b = 4320$ | $S(6,2)$ |
| | 2-$(36,16,1452), b = 7623$ ∗ | $S(6,2)$ |
| | 3-$(36,18,768), b = 6720$ ∗ | $S(6,2)$ |
| $[45,14,12]_2$ | 2-$(45,12,8), b = 120$ | $U(4,2):Z_2$ |
| $[45,15,12]_3$ | 2-$(45,12,3), b = 45$ | $U(4,2):Z_2$ |

TABLE 14. $t$-designs constructed from codes.

**Remark 5.1.** *To the best of our knowledge designs indicated by $*$ have not been known before, so we proved the existence in terms of parameters. For further information about all other designs see Mathon and Rosa (2007) and Crnković* et al. *(2016).*

| $\mathscr{C}$ | SRG $\Gamma$ | Aut($\Gamma$) |
|---|---|---|
| $[28,7,12]_2$ | $\Gamma_1 = (63,30,13,15)$ | $S(6,2)$ |
| $[252,21,36]_2$ | $\Gamma_2 = (378,52,26,4)$ | $S_{28}$ |
| $[336,21,96]_2$ | $\Gamma_3 = (630,68,34,4)$ | $S_{36}$ |
| $[36,15,8]_2$ | $\Gamma_4 = (135,64,28,32)$ | $O^+(8,2):Z_2$ |
| $[45,14,12]_2$ | $\Gamma_5 = (45,12,3,3)$ | $U(4,2):Z_2$ |
| $[45,14,12]_2$ | $\Gamma_6 = (40,12,2,4)$ | $U(4,2):Z_2$ |
| $[36,8,14]_2$ | $\Gamma_7 = (36,14,4,6)$ | $U(3,3):Z_2$ |
| $[120,8,56]_2$ | $\Gamma_8 = (120,56,28,24)$ | $O^+(8,2):Z_2$ |

TABLE 15. SRGs constructed from codes.

**Remark 5.2.** *Two graphs $\Gamma_2$ and $\Gamma_3$ are triangular graphs, $T(28)$ and $T(36)$, respectively. The SRGs $\Gamma_5$ and $\Gamma_6$ are completely classified, rank 3 graphs (see Brouwer 2007). Graphs $\Gamma_4$ and $\Gamma_8$ are the complementary graphs of the polar graph $O^+(8,2)$ and $NO^+(8,2)$, respectively. Strongly regular graphs $\Gamma_1$ and $\Gamma_7$ can be constructed from symmetric incidence matrices with all-one diagonal of the symmetric block design with parameters* $(36,15,6)$ *and* $(63,31,15)$.

| $\mathscr{C}$ | DRG $\Gamma$ | Aut($\Gamma$) |
|---|---|---|
| $[63,21,9]_2$ | $v = 63$, [6,4,4,1,1,3] | $U(3,3):Z_2$ |
| $[36,15,8]_2$ | $v = 135$, [14,12,8,1,3,7] | $S(6,2)$ |
| $[36,8,14]_2$ | $v = 56$, [27,16,1,1,16,27] | $Z_2 \times O(7,2)$ |
| $[36,8,14]_2$ | $v = 56$, [27,10,1,1,10,27] | $Z_2 \times O(7,2)$ |

TABLE 16. DRGs constructed from codes.

**Remark 5.3.** *DRG with $v = 63$ is graph with diameter 3, primitive distance-regular graph well known as generalized hexagon of order (2,2). The graph with $v = 135$ is the unique distance-regular graph with this parameters, primitive distance-regular with diameter 3. The two graphs having $v = 56$ are antipodal (but not bipartite) distance-regular graphs with diameter 3. For more information about constructed DRGs we refer the reader to the book of Brouwer* et al. *(1989) and the article of Dam* et al. *(2016).*

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