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COMPLEX FLUID FLOWS: CLASSICAL FLUID MECHANICS VS. THERMODYNAMICS OF FLUIDS

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ABSTRACT. A comparison between the characteristics of fluid flows which were dealt with by classical mechanics of fluids and the features of complex materials presenting particular fluid properties subject to complicate conditions is performed. The fluids and their motion are analyzed and the mathematical models describing them are minutely shown. This analysis allows us to motivate why the most appropriate mathematical description of complex fluid flows must be the concern of thermodynamics of fluids. A parallel between the laminar and turbulent regimes of motion is also presented whereas the laminar/turbulent transition is associated with dynamics which at its last stages turns into a turbulent motion. In this way, an explanation of the transformation of perturbations into fluctuations is offered.

1. Introduction

In classical fluid mechanics the concept of fluid was heuristical, doubled by a physical description. Accordingly, physically, fluid could be a material in the aggregation state of a gas, liquid, vapors. A further description was provided by the so called classical thermodynamical state equation, *e.g.*, the law of perfect gases $p = \rho RT$ (where R is the gas constant), relating the pressure p, the density ρ and the temperature T at rest. Moreover, a Fick type relation $\mathbf{q} = -D\nabla T$ was imposed, connecting the heat flux $\mathbf{q}$ by the gradient of temperature; and a relation expressing the dependence of the internal energy e on the temperature, *e.g.*, $e = c_v T$ (where c_v is the latent heat at constant volume) was considered too.

These physical characterizations were supplemented by heuristical-mechanical description, *e.g.*, a liquid is a substance which takes the form of the recipient in which it is put. In addition, it is characterized by the fact that at a rest a fluid opposes no resistance (*i.e.*, internal friction) to its deformation (for instance when a solid is immersed in it). This heuristical-mechanical characterization was first formalized by Newton more than 300 years ago in a particular form

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} ,$$

where τ_{xy} is the shear (viscous) stress, μ is the coefficient of dynamic viscosity, $\frac{\partial u}{\partial y}$ is part of the tensor of velocity of deformation. The Newton law was derived experimentally and is the first attempt to formalize the internal friction in fluids in a close analogy with the dry friction: the resistance (friction force) undergone by a body at a surface of contact is proportional to the normal force (weight) acting on the surface. Inside the fluids the surface of contact is situated between fluid layers and the normal force is the normal gradient $\left(\frac{\partial u}{\partial y}\right)\mathbf{j}$ while the motion is characterized by the velocity $u\mathbf{i}$, with $\mathbf{i}$ and $\mathbf{j}$ the unit vectors along the x and y axes. The coefficient μ is a complex physical quantity.

Latter on, Stokes formalized the mechanical definition of a fluid by means of six postulates all of them leading to a relationship between the shear stress and the velocity of deformation tensor (in fact, its symmetric part). In spite of the fact that a fluid was characterized not only by this mechanical relation, but also by the physical relationship like, for instance, the law of perfect gases, the Fick type law and the internal energy law, the classical fluid dynamics was viewed as a mechanical science. The quoted physical relationships were used but not considered as organically connected with mechanical characterization. This followed by the fact that the temperature was not a mechanical quantity. In addition, in specific circumstances the mechanical equations (of mass and momentum conservation) were decoupled by the energy equation and this equation was studied no longer. Even in situations when the kinetic energy was considered, the terms containing the temperature were looked at as simply additional effects given by a non-mechanical science.

In classical mechanics the prototypes of fluids were water and air. This situation dramatically changed in the first past half-century when “new materials”, which hardly can be associated with the traditional idea of fluids, were included in the class of fluids: melting metals, mayonnaise, mud, blood, plasma, magma, sand, asphalt, flour in big tanks, sets of stars, sets of cars moving on a highway, micropolar and non-Newtonian fluids, most of them synthetic substances. They were described by several constitutive laws, not only by the “mechanical” Stokes law but also by constitutive laws like Fick law or internal energy law. In addition, the constitutive laws were the most general possible obeying some general requirements (*e.g.*, objectivity principle). Correspondingly, the material conceived as a fluid was defined mathematically by means some transformations invarianting the constitutive equations.

Accordingly, the framework of classical fluid mechanics was to be enlarged up to the thermodynamics of fluids as a part of thermodynamics of processes far from equilibrium.

2. Flow

Correspondingly, the mechanical flow of fluids was appropriate no longer since the complicated nontraditional fluids can undergo complicated phenomena. The mechanical motion could not be separated by physical, chemical, etc. changes, because the influence on the mechanical motion was so important that the fluid motion was to be considered as a more general dynamics involving phase space functions describing not only mechanical phenomena $(\mathbf{v}, p)$, thermomechanical phenomena $(\mathbf{v}, p, \rho, T)$, but also electromagnetic phenomena $(\mathbf{v}, p, \rho, T, \mathbf{E}, \mathbf{H})$ etc. The motion itself was to be interpreted no longer in the physical space but in the phase space.

In addition, more complex situations arising in reality (nature, industry, experiments, etc.) claimed for the study of fluid motion, *e.g.*, the blood flow in elastic vessels, the turbulent flow of the atmosphere where the water in the liquid, solid (ice) or vapors form coexisted and were continuously transforming one into each other.

All these situations of complicated flows of complex fluids in complicated situations (surroundings) imposed thermodynamics of fluids as the only science able to describe all these unusual facts from the point of view of the traditional fluid mechanics. Consequently, thermodynamics of fluids treats complex flows of complicated fluids during their phase transitions, in presence of elastic boundaries of thermal, magnetic and electric fields. This means that the science governing the dynamics of complex fluids which are thermally, electrically conductors and are responding to magnetic influences is more physical than mechanical. In addition, if the fluid is a biological substance, then biological phenomena are described suitably by the framework offered by thermodynamics of fluids. This means that the modern concept of physics is more or less that of Aristotel and includes any phenomena ranging from mechanical to those which take place in life systems, economics, society and linguistics.

The study of fluid flows is of interest to: the knowledge (chaos, turbulence in any phenomenon); engineering (aviation); naval design (propellers); nonconventional energy (windmils); oil extraction (in seas); physics (premodelling, scale of motion, constitutive equation, state equations); astrophysics; meteorology (dynamics of atmosphere); hydrology (dynamics of rivers, shallow waters waves in lakes); hydraulics (canalization systems of towns and villages, water, air and a large variety of “oils” (non-Newtonian fluids in engines and other confinements); lubrication (related to the reduction of dry friction in relatively moving pieces of a mechanism); combustion (related to car functioning); oceanography (marine and oceanic currents (streams), solitary waves (like tsunamics), deep water waves); volcanology (magma motion); nuclear technology (Tokamaks containing plasma rings); chemistry; biology (circulatory systems, cancer, enzymes); mathematics (for new problems and methods inspired by them).

In the following we are concerned with complex fluid flows in complicated situations mainly of interest to engineering applications.

3. Mathematical models

Mathematical models governing these fluid flows consist of: sets of balance equations (corresponding to physical laws of motions in fluid system), constitutive equations (here included equations of state, representing the mathematical description of the fluid material, boundary and/or initial conditions, liberating the fluid system of its ties with the environment and/or of its past); other restrictions of physical nature (*e.g.*, positivity of density); the class of the solutions (problem).

In a mathematical model we distinguish: independent variables (time and/or space coordinates); unknown functions (phase space functions characterizing the dynamics); data (*e.g.*, parameters, initial data, boundary data). Domain of motion is the geometric locus of the fluid. Let it be $\Omega \subset \mathbb{R}^n$, $n = 1,2$ or 3. It can be: bounded (in containers, confined plasma, circulatory blood system, flow in a closed windtunnel or watertank etc.); unbounded (entire $\mathbb{R}^n$) in presence of unbounded boundaries (flight near the Earth, motion in ocean water); external to some bodies (*e.g.*, airplanes in an infinite stratum of air (an idealization appropriate where the aircraft flies at a high altitude), a wing of an aircraft or a section of it (airfoil) in the same conditions, submarines in oceans (also idealizing the ocean as an unbounded domain)); with free (a priori not known) boundaries (flows in elastic tubes, jets, wakes behind bodies, navigation with partial submersion in water, on inclines planes, bubbles).

The models of fluid dynamics are investigated *theoretically*, *numerically* and *experimentally*. The most trustful are the last ones.

Mathematical study of these models contains: models derivation, approximations of models, simplification of models, quantitative and qualitative analysis of the solution and solution set, respectively. Quantitative analysis has as goal the determination of the solution as function of the independent variables for fixed data. This determination can be exact or approximate, theoretical or numerical. Qualitative (geometric) analysis deals with the determination of the solution set (not only of a solution) in dependence on data. It concerns: dependence on initial data (continuous dependence, Liapunov stability, asymptotic stability, attractivity, sensitive dependence on initial data); dependence on parameters (bifurcation); dependence on other data; asymptotic behavior of the solution with respect to some independent variable(s); existence, uniqueness and regularity of the solution in dependence on data.

In mathematical nondimensional models of complex fluid flows several *parameters* occur. *Physically* each of them is related to an effect (thermal, electrical, magnetic, of compressibility, rotation, porosity) and occurs in the governing equations as a coefficient of the corresponding terms. *Mathematically* a parameter is a variable with respect to which no differentiation or integration is performed; therefore a parameter is not an independent variable. Usually the parameters represent the ratio of two characteristic forces or other quantities (length, time). The ratio of two lengths is referred to as an *aspect ratio*. We quote a few examples of parameters: Reynolds ($\mathscr{R}e$), Rayleigh ($\mathscr{R}a$), Prandtl ($\mathscr{P}r$), Rossby ($\mathscr{R}o$), Boussinesq ($\mathscr{B}o$), Mach ($\mathscr{M}a$) numbers, where $\mathscr{R}e$ is the ratio of the inertia and viscous forces (we mean the most important terms of these forces), $\mathscr{M}a$ is the ratio of the velocity of the fluid and of the sound etc. In addition in a non-dimensional model, the parameters are the only ones to remind the physics of the phenomenon, because all independent variables and unknown functions have no dimensions, they are abstract mathematical quantities. Finally, let us mention that the set of all parameters in a model ensures the similarity of all phenomena which are characterized by these parameters. For instance, if two models contain only the Reynolds number and it takes the same value in both of them, then the two fluid flows governed by these models are mechanically similar. Remind that $\mathscr{R}e = UL/\nu$, where U and L are the characteristic velocity and length and ν is the coefficient of kinematic viscosity. Assuming that the two fluids are characterized by the same ν, it follows that their (mechanical) motions are similar if the velocity of the first is k time larger than of the second, while the ratio between their velocities is inverse. This kind of similarity differs from the geometric homotety (*e.g.*, two ellipses are homotetic if their eccentricities are the same; in general a figure is similar (homotetic) to other if one is obtained by the other by a homotety defined by a geometrical similarity parameter). The mechanical, thermal etc. similarity ensures that a phenomenon observed in reality is the same, from the mechanical, thermal etc. point of view, as a phenomenon occurring in an experiment.

Particular classes of solutions lead to *simplified models*. Among them we quote: solutions not depending (in an explicit way) on one or several independent variables. If they do not depend on time, they are called steady or stationary; if they do not depend on one or two space variables they are called two-dimensional or one-dimensional, respectively; if they do not depend on any space variable then they are called homogenous; if they do not depend at all on the independent variables, then they are called uniform; other particular types of solutions are the waves (*e.g.*, normal modes (in convection studies), solitons (in particular tsunamies)); solutions in the form of Fourier series; solutions in the form of asymptotic series. The Fourier series solutions depend on some independent variables in a known way, leading to simplified models. The solutions in the form of general Fourier series upon a total set in the space of the problem reduce the solving of the problem to the determination of the coefficients. The asymptotic series with respect to some independent variable(s) about on accumulation point lead to simplified models which do not depend on that variable(s) any longer.

The *methods* used in thermodynamics of fluids differ from a branch of this science to other. Among them we quote: analytical, geometrical, topological, algebraic, statistical and asymptotical methods to name a few of them. For the case of complex fluid flows the asymptotic methods with respect to a small parameter ε are crucial. They are called perturbation methods and are grouped in two classes: methods of regular perturbations and methods of singular perturbations. The first apply when the solution has a unique order of magnitude throughout the domain of all independent variables, while the second type of methods apply when in the space of the independent variables there is at least one thin layer, referred to as the *boundary layer*, across which the solution undergoes a fast variation in order to pass from an order of magnitude to other. In the first case the solution is written as a unique asymptotic expansion with respect to the asymptotic sequence $1, \varepsilon, \varepsilon^2, \cdots$ as $\varepsilon \to 0$ and it is introduced in the model and at each order the corresponding model of asymptotic approximation is deduced. In the second case at least two asymptotic expansions of the solution are used to derive models of asymptotic approximation inside the boundary layer and outside it. In a model of asymptotic approximation all terms have the same order of magnitude unlike the situation with the given model. This is why numerical methods are applied not to the given model but to its models of asymptotic approximation.

In fluid mechanics $\varepsilon = (\mathscr{R}e)^{-1/2}$ and the boundary layer is formed as $\varepsilon \to 0$, *i.e.*, $\mathscr{R}e \to \infty$ ($\nu \to 0$). If the boundary layer occurs in a model describing a mechanical phenomenon, then it is called a dynamic boundary layer, if it occurs in a model governing the heat transfer, then it is referred to as a thermal boundary layer, in a model describing the atmospheric dynamics it is called the atmospheric boundary layer, in a turbulent flow it is referred to as a turbulent boundary layer etc. For applications the importance of the boundary layer is enormous: it explains the separation of the fluid from the surface of an immersed obstacle in fluid and permits to compute the drag.

The first boundary layer was conceived and found in fluid mechanics by Ludwing Prandtl in 1904. He proposed one of the most important method of asymptotic approximation for singular perturbation problems (Georgescu 1995).

The *class of a model* consists of the domains for all independent variables, unknown functions and data. In the case of the unknown functions in the definition of the domain it is specified the smoothness (regularity), discontinuity etc. Correspondingly the solutions can be classical (regular), generalized, chaotic (irregular), shock waves (discontinuous), random variables, measures of probability etc.

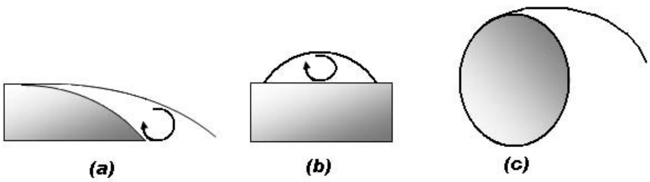

FIGURE 1. Recirculating flows: (a) slender form; (b) reattachment; (c) boundary layer.

3.1. Experiments and visualization (of primary interest for engineers).

One of the most famous syntheses of fluid flows from an experimental point of view is to be found in the album book by Van Dyke (1982). It concerns creeping flows, lamina flows, separation, vortices, instability, turbulence, free-surface flows, natural convection, subsonic flows, shock waves, supersonic flows (see the bibliography). Van Dyke (1982) presented photographs of the spectrum of fluid flows (*i.e.*, stream lines), realized by special photographic techniques, produced by means of smoke, dye, small suspended particles, hydrogen bubbles, electrolytic precipitation in water etc. We also mention other techniques to measure fluid flow characteristics, *e.g.*, hot wire anemometry. Van Dyke (1982) presented:

- *flows past* (about, around) flat plates, cylinders, cones, ogives, bullets, airfoils, wings (near leading and trailing edges), airplanes, projectiles;
- *flows in/from* nozzles, orifices, converging or diverging tubes, in long tubes, jets, plumes, from a wedge in a shock tube, on an accelerated plate;
- *flows behind bodies* (wakes), vortex sheets (behind a cylinder in (for field) subsonic flow or behind a ship, steps, plates at a nonnull attack angle, rotating propellers, rectangular ring (initially it is laminar and then becomes turbulent), trailing vortex from the tip of rectangular ring;
- *impulsively starting moving bodies* plates, foils, cylinders, plumes;
- *Bénard convective cells* of honeycomb form (others are cloud streets);
- *bubbles* in air rising in a fluid.

Furthermore, the cavitation phenomenon, which destroys naval and marine propellers, is shown; the influence of the suction on the bubble formation on various wings, behind them in subsonic or supersonic flow, is revealed; *bores* (traveling forms of hydraulic jumps), *waves* (capillarity waves, gravity waves, atomization from a nozzle, shallow water waves, deep water waves), *recirculating flows* on wings, airfoils, metropolitan trains (Fig. 1b) depending on their blunt or *slender form* (Fig. 1a) and attack angle, the separation (Fig. 1) and *reattachment* (Fig. 1b) of the *boundary layer* (Fig. 1c) are shown; the influence of suction, rugosity, attack angle, various forces and especially of the Reynolds number on the flows are shown.

In Fig. 2 the spectrum of the flow behind a cylinder, as $\mathscr{Re}$ is increased, is sketched from the left to the right. First the flow presents *no vortices* (Fig. 2a), then *a vortex appears* behind and other appears ahead the cylinder (Fig. 2b), then *the vortices increase* (Fig. 2c), then *several vortices are formed* (Fig. 2d), for instance the famous Kármán vortex street (Fig. 2e), then the formation of a turbulent wake (Fig. 2f). *Vortices*: vortex breakdown above a triangular wing (Concorde airplane, instability of the vortex line, downwards becomes

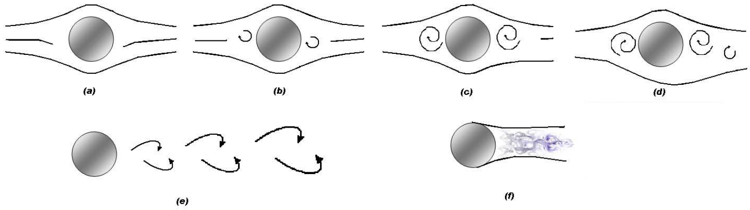

FIGURE 2. Spectrum of the flow behind a cylinder: (a) no vortices; (b) a vortex appears; (c) the vortices increase; (d) several vortices are formed; (e) the famous Kármán vortex sheet; (f) a turbulent wake.

turbulent), spiral vortices on a spinning disk (instability of a rotating motion), spiral vortices on a cone rotating in a stream (instability), vortices behind an inclined slender body, vortices behind an inclined triangular wing, horseshoe vortices sheet on a cylinder in a boundary layer, Kármán vortex street behind a circular cylinder at various $\mathscr{R}e$, smoke at various levels in a vortex sheet, turbulent vortices (eddy vorticity).

In laminar flow, the *vorticity field* (rot $\mathbf{v}$) is uniformly distributed, the emergence of vorticity filaments implying the turbulent birth. In inviscid fluid flows a vortex is a bounded region in a domain of motion where $\mathrm{rot}\mathbf{v} \neq 0$. Experimentally the vorticity is a superposition and juxtaposition of vortices of various scales; this takes place in a turbulent regime.

4. Laminar vs. turbulent flows

Unlike in a laminar regime, in the turbulent regime the unknown functions are random. Experimentally an *instantaneous velocity field* is obtained by a short time of exposure of the photography. Mathematically it is the usual field while a *mean motion* is characterized by average velocity field, which, experimentally is obtained after a long time exposure. For the same conditions, Van Dyke (1982) showed instantaneous and mean flow spectra in a turbulent regime. They reveal the fact that in turbulence the physically relevance is the mean flow and not the instantaneous flow, as observed by Osborne Reynolds as early as 1883, because the characteristics of the instantaneous flow vary around these of the mean flow. He proposed the splitting of the instantaneous flow characteristics, *e.g.*, $\mathbf{v}(t,\mathbf{x})$ into mean (averaged) characteristics $\bar{\mathbf{v}}(t,\mathbf{x})$ (obtained by applying a probabilistic mean to $\mathbf{v}$) and a *fluctuation* $\mathbf{v}' = \mathbf{v}(t,\mathbf{x}) - \bar{\mathbf{v}}(t,\mathbf{x})$. The mean is a nonrandom function, while the instantaneous field and the fluctuation are random. The mean of the fluctuation characteristics is equal to zero, while the mean of their products is not. This is why the turbulence level (degree, intensity) is defined as the following mean:

$$\sigma = \frac{100}{U}\sqrt{\frac{1}{T}\int_0^T \frac{u'^2+v'^2+w'^2}{3}dt}\ ,$$

where $\mathbf{v}' = (u', v', w')$ is the fluctuation velocity, $\bar{\mathbf{v}} = (U, 0, 0)$ is the mean velocity (we considered a particular flow) and the time T of temporal averaging is much larger than the time of any deviation of the fluctuations from the mean flow. We introduce

$$\mathbf{v} = (v_1, v_2, v_3)\ , \quad \bar{\mathbf{v}} = (\bar{v}_1, \bar{v}_2, \bar{v}_3)\ , \quad \mathbf{v}' = (v'_1, v'_2, v'_3)\ , \quad \mathbf{x} = (x_1, x_2, x_3)\ .$$

It is unanimously admitted that in the turbulent regime the (instantaneous) Navier-Stokes equations are still valid. Let us write them in the dimensional form, writing, for convenience,

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3} = 0\ , \tag{1}$$

$$\frac{\partial v_i}{\partial t} + (\mathbf{v} \cdot \mathrm{grad}) v_i = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \Delta v_i\ , \qquad i = 1,2,3, \tag{2}$$

for the sake of simplicity, we considered the incompressible case. We take the mean of them to obtain the so-called equations of the mean flow

$$\frac{\partial \bar{v}_1}{\partial x_1} + \frac{\partial \bar{v}_2}{\partial x_2} + \frac{\partial \bar{v}_3}{\partial x_3} = 0\ , \tag{3}$$

$$\frac{\partial \bar{v}_i}{\partial t} + (\bar{\mathbf{v}} \cdot \mathrm{grad}) \bar{v}_i = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \Delta \bar{v}_i + \frac{\partial(-\overline{v'_i v'_j})}{\partial x_j}\ , \qquad i = 1,2,3. \tag{4}$$

We subtract (3), (4) from (1) and (2) respectively to obtain the equations of the fluctuation

$$\frac{\partial v'_1}{\partial x_1} + \frac{\partial v'_2}{\partial x_2} + \frac{\partial v'_3}{\partial x_3} = 0\ , \tag{5}$$

$$\frac{\partial v'_i}{\partial t} + (\mathbf{v}' \cdot \mathrm{grad}) v_i + (\bar{\mathbf{v}} \cdot \mathrm{grad}) v'_i = -\frac{1}{\rho} \frac{\partial p'}{\partial x_i} + \nu \Delta v'_i\ , \qquad i = 1,2,3. \tag{6}$$

In general, by splitting the instantaneous fields into mean and fluctuation the equations (1)-(2) are equivalent to the union of two coupled systems (3), (4) and (5), (6). Since the new system (3)-(6) is difficult to solve, several ideas were proposed. The first one is to transform the system (3)-(4) in a system where only mean flow characteristic occurs. Indeed, equations (4) contain the characteristics of the mean flow $(\bar{\mathbf{v}}, \bar{p})$ as well as of the fluctuation $\frac{\partial(-\overline{v'_i v'_j})}{\partial x_j} = R_{ij}$, where R_{ij} are the components of the Reynolds symmetric *stress tensor* (measuring the global effect of the deviation of the velocity $\mathbf{v}$ with respect to the mean $\bar{\mathbf{v}}$). In spite of the occurrence of the Reynolds tensor, the equations are referred to as the mean flow equations because all other terms in them are characterizing the mean flow. In order for the equations (4) contain only characteristics of the mean flow a method is to assume that R_{ij} expresses in terms of these characteristics. There are a lot of relationships between R_{ij} and $\bar{\mathbf{v}}$ of various forms (algebraic, differential, integral), deduced from experiments. The most simple is

$$R_{ij} = 2\nu_t D_{ij}\ , \tag{7}$$

where ν_t is the turbulent viscosity, which can be constant (Boussinesq idea), of the (Prandtl) form (similar to the laminar case) $\nu_t = l^2 \frac{\partial \bar{v}_1}{\partial x_2}$, l being the mixing length, and many other forms. The model obtained by introducing (7) into (4) is called a *turbulence (naïve) model.*

Another idea is to obtain another system in $\bar{\mathbf{v}}$ and R_{ij} such that together with (1) and (2) form a closed system (the number of equations and the unknown functions are the same). It

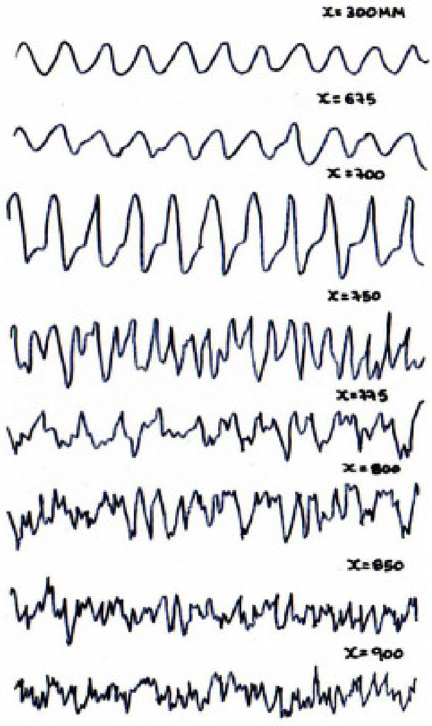

FIGURE 3. Typical oscillograms of pulses in transition without turbulent spots.

seems that this new system can be obtained by multiplying (2) by v_k and then taking the mean of the resulted system. However, it is immediately seen that in the new system means of three terms products of the form $\frac{\partial(-\overline{v'_i v'_j v'_k})}{\partial x_j}$ are present such that the system (1), (2) cannot be closed by supplementing it with the mean part of $\overline{v_k^{(2)}}$. Going on with the same procedure it follows that it is impossible to get a closed system in the mean flow characteristics. In view of these facts, relation (7) is referred to as the *closure relation*.

In some concrete situations Van Dyke (1982) reported the experiments that revealed the strong dependence of the spectrum of the fluid flows on $\mathscr{R}e$. This follows from the fact that in the nondimensional form of the Navier-Stokes model governing these flows depend on $\mathscr{R}e$. Indeed, in the incompressible case, in the domain Ω of boundary $\partial\Omega$ this model reads:

$$
\begin{aligned}
&\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot \operatorname{grad})\mathbf{v} = -\operatorname{grad} p + \frac{1}{Re}\Delta \mathbf{v},\\
&\operatorname{div} \mathbf{v} = 0\ ,\\
&\mathbf{v}|_{t=0} = \mathbf{v}^0\ ,\\
&\mathbf{v}\big|_{\partial\Omega} = \mathbf{w}\ ,\\
&\lim_{|\mathbf{x}|\to\infty} = (U,0,0)\ ,
\end{aligned}
$$

where $\mathscr{R}e = UL/\nu$, L being a characteristic (reference) length. Especially as $\mathscr{R}e$ increases very much, the stability of the existing basic fluid flow is lost on the account of the bifurcation of a new solution of the Navier-Stokes model and the vorticity field becomes more complicated. The continuous formation of new motions and especially of vortices of various scales is possible only if the perturbations take energy from the basic flow, at the end the flow becoming turbulent and the perturbations becoming fluctuations. This is why, the instability and turbulence set in at high $\mathscr{R}e$.

Instability is one among the causes of emergence of turbulence (see Fig. 3) (Georgescu 1985, 2009, 2017). However, the mechanisms determining the birth (nature, origin) of turbulence remains the greatest unsolved problem of fluid mechanics. No answer is given to the questions: How perturbations are turning into fluctuations? How to establish a threshold at which an ordinary function becomes random? Turbulence is a property of the flow and not of the fluid. It is a complex (and not mechanical) phenomenon, the most important feature of which is the randomness. This random feature must be taken into account in modelling from the very beginning even if we postulate that the Navier-Stokes model holds in turbulent regime too.

The most complex is the atmospheric turbulence. In it the dimension of the vortices rang from 1 m to several thousands kilometers. A heuristical definition reads: turbulence is a motion described relevantly by random quantities and not by instantaneous fields. The instantaneous fields cannot be determined by experimental data. This means that the same (initial, boundary) conditions lead to various instantaneous fields any time the experiment is repeated.

In spite of the fact that we do not know what turbulence is, it is possible to characterize mathematically very accurately particular turbulent motions. Among them we quote free turbulence (*e.g.*, in jets, wakes) and wall turbulence (*e.g.*, near the bodies, the boundary layer). Turbulence can have several stages but, usually, by turbulent motion we understand a *fully developed turbulence*, *i.e.*, at every point of the domain of motion the characteristics are turbulent (random).

The turbulence (Van Dyke 1982) occurs in jets at far-field supersonic and hypersonic flows, wakes (behind bodies as for instance projectiles and riffle bullets), in some boundary layers, shock wave interaction. The laminar motion cannot be defined either mathematically as well as physically. We can characterize it experimentally. Table 1 reports a comparison between the features of the two main regimes of motion: laminar and turbulent.

5. Transition vs. deterministic chaos

By transition we mean the intermediate regime between the laminar and turbulent regimes. It has three stages, the last one, the preturbulent stage, characterized by the deterministic chaos. Transition is different in different geometric configurations, but there are two large classes of transition: in the first the laminar coexists with turbulent zones while in second not. The flows on a flat plate (let $\mathscr{R}e = Ux/\nu$, where x is the distance of the leading edge) belong to the first class. Then if at the leading edge sinusoidal oscillations are introduced into the boundary on the flat plate, then as x is increased the oscillations become more and more complicate, finally become random); the laminar sublayer of a turbulent boundary

TABLE 1. Comparison between the laminar and turbulent regimes of motion.

Laminar regime	Turbulent regime
regular motion	irregular (chaotic) motion (different from deterministic chaos)
smooth trajectories	irregular trajectories (either in the physical or in the phase space). They are extremely complex and sinuous as if they do not obey any law.
predictable motion	unpredictable (instantaneous) motion
physically relevant instantaneous quantities	physically irrelevant instantaneous quantities but relevant mean quantities
motion of molecular scale	motion of molar scale
uniformly distributed rot **v**	nonuniformly distributed vorticity field rot **v**
$\mathbf{v}(\mathrm{t},\mathbf{x}) \in \mathbf{R}^n,\ n = 1,2,3$	$\mathbf{v}(\mathrm{t},\mathbf{x}) \in \mathbf{R}^3$ and $\mathbf{R}^2$ in some special situations
stationary or nonstationary motion	only nonstationary motion (but can exist (statistically) stationary or uniform mean motion)
existence and uniqness of the solution	nonuniqness of the (instantaneous) solution
the drag is proportional to $\lvert\mathbf{v}\rvert$	the drag is proportional to $\lvert\mathbf{v}\rvert^2$, implying more fuel consumption for eddies formation and transformation than in the laminar regime; as a consequence turbulence avoided at the croisière flight of aircrafts
no mixing is present	strong mixing (whence the usefulness in chemical industry)
homogeneous characteristics of the motion can exist	homogeneous instantaneous characteristics of the motion cannot exist but homogeneous mean ones can exist

layer of a subsonic ($\mathscr{M}a < 1$), transonic ($\mathscr{M}a \sim 1$), sonic ($\mathscr{M}a = 1$), supersonic ($\mathscr{M}a > 1$) and ipersonic ($\mathscr{M}a \gg 1$) far-field flow; some turbulent spots are forming insulated in a surrounding turbulent region and these spots come down, at a fixed point giving rise to the intermittence phenomenon (*i.e.*, the alternance of turbulent and laminar regimes). The flows in tubes belong to the second category: at a given time, throughout the tube there is a single type of flow.

The passage to turbulence is longer or shorter in dependence on flow configurations. Anyhow as $\mathscr{R}e$ is increased the laminar regime turns into transition and, finally, to turbulent regime, with some stages longer or shorter, depending on several causes, *e.g.*, suction, rugosity of the walls, type of fluid (in a series of papers (Georgescu 1995) on stability arguments we try to explain the Toms effect: the drag reduction by additives supply (in order to reduce the pressure losses) considerably reduces the energy of the turbulent flow in comparison with the solvent (nonadditivated liquid). Van Dyke (1982) reported a graduate transition in a jet from a jet (laminar, transition, turbulent, ejection of spray droplets).

In some transition regimes the Mach number $\mathscr{M}a$ (equal to the velocity of the fluid flow/sound speed) has an important influence. This happens when the fluid is the air at various velocities. We recall that if the velocity of the air over passes one third of the sound speed, then the air is no longer modelled as an incompressible but a compressible fluid and in the equations of motion the Mach number occurs.

The deterministic chaos is a phenomenon uniquely (and so deterministic) determined by the equations and the initial data but it has only physically relevant characteristics. Therefore it possesses a feature of the laminar flow (uniqueness) and other of the turbulent flow (relevant means). The deterministic chaos is a preturbulent and not a turbulent flow. Lorenz (1963) was the first to formulate the concept of the deterministic chaos. By that time as an applied mathematician he investigated the simplified equations deduced from the Navier-Stokes equations and intended to describe the dynamics of the atmosphere. He looked for the solutions in the form of Fourier series and truncated them at the first terms. In this way the only unknown functions were the amplitudes and they depended only on time. These amplitudes formed the solution of the Cauchy problem for the (now called) Lorenz equations:

$$\begin{aligned} \dot{X} &= -\sigma X + \sigma Y \ , \\ \dot{Y} &= rX - Y - XZ \ , \\ \dot{Z} &= -bZ + XY \ , \end{aligned}$$

where the dot over the quantities stands for the time derivatives and r, σ, b are three parameters. He fixed σ and b and left r vary, finding different attractors for different values of r. These attractors became more and more complicate to attain the limit $r \approx\ 24$ at which a chaotic attractor was born. The corresponding phase dynamics (trajectories) were irregular, complicated, the motion became unpredictable in the long run, depended sensitively on initial data rending the system not appropriate to numerical computations and the power of the computer was irrespective.

Whence the crucial importance of modeling and studying nonlinear dynamics and bifurcation. Subsequently more and more chaotic dynamics were reported in the literature and, so, a new paradigma of our times emerged: the deterministic chaos is intrinsic (internal) property of the system. This behavior arises in almost all systems and it is born without any infusion of randomness (*e.g.*, white noise) into the system. It is a bridge between laminar and turbulent motion, providing an answer how turbulence sets it: the motion first possesses a deterministic chaotic behavior which further degenerates into turbulence. Even the simplest systems can present a deterministic chaos (the continuous dynamical systems must have at least the dimension equal to three while in the case of discrete dynamical systems even the dimension one suffices). An idea of the sensitive dependence on initial data is given by billiards while the concept of unpredictability (in the long run) is understood from the graph of $y = \sin \frac{1}{t}$ (see Fig. 4).

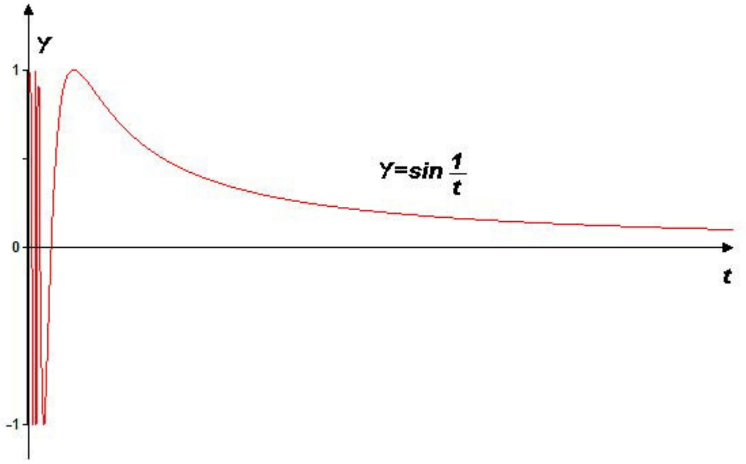

FIGURE 4. Graph of $y = \sin\frac{1}{t}$ representing the concept of unpredictability.

Editor's note

Prof. Adelina Georgescu, a former corresponding member of the "Accademia Peloritana dei Pericolanti", passed away on May 1st, 2010.[†] *She delivered this lecture on the occasion of a meeting entitled "Complex Fluid Flows", which was held at the University of Messina in 2007. Prof. Liliana Restuccia, a close collaborator of the author, has taken care of the review and editing of the draft manuscript that Prof. Adelina Georgescu had originally prepared for the meeting and that is published posthumously in grateful memory of a highly respected, profoundly generous colleague.*

[†] Rocşoreanu, C. (2011), *Annals of the Academy of Romanian Scientists. Series on Mathematics and its Applications* **3**(1), 3−7.

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