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MODELS OF ASYMPTOTIC APPROXIMATION IN ATMOSPHERE DYNAMICS

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ABSTRACT. In applying a method of regular or singular perturbations, some information additional to that contained in the studied model is necessary. Moreover, the order of magnitude of the unknown functions, data and independent variables must be known too. All these are given by practicians working in the field. We exemplify these ideas on two models governing the motion of the atmosphere.

1. Structure of the atmosphere

The atmosphere of the Earth is far from being a simple and homogeneous fluid (Georgescu 1992a,b). Indeed, the atmosphere is a *geophysical fluid* subject to astronomic (*e.g.*, terrestrial) influences. It is a mixture of: gases of concentration strongly varying with height; water in the form of vapors, liquid and solid (glace); aerosols (suspensions, *e.g.*, dust, other particles, pollutants); ozone. Hence, the atmosphere is a *micropolar multiphase fluid*. In it phase transition occurs, therefore it is a non-Newtonian fluid. Consequently, the dynamics of atmosphere can be appropriately treated in the framework of thermodynamics of processes far from equilibrium.

Among the actions on the atmosphere we quote: the electric and magnetic field of the Earth; inertia forces due to the motion of the Earth (we recall that the centrifugal force added to the gravitational field results in the gravific field, Coriolis force is acting since the fluid has a fast motion; mareic forces due to the Moon and the Sun); the (approximately) spherical form of the Earth; orography (relief, repartition of the ocean and land); electromagnetic radiation of Sun; solar wind; volcans; cometae; meteorits; cosmic radiation. All these actions produce very rich equations of momentum and energy, *i.e.*, these equations have many terms. The unknown functions occurring in them strongly vary with respect to the height. This means that the general mathematical model governing the atmosphere dynamics is particularly complex (Georgescu 2009, 2017). This is the reason why this model is approximated according to the order of magnitude of the terms describing various physical, mechanical, chemical etc. effects.

According to the main distinct thermodynamic processes taking place in atmosphere we distinguish three strata. The most investigated is the *troposphere*, which is situated near the

Earth. The troposphere is much influenced by the thermal state of the land. It is conceived as a thermal engine ensuring the transport of the heat received from the Sun from the Equator to the poles, the thermal gradient varying with the seasons (during the winter being larger). The troposphere has as a substrate the atmospheric Eckman boundary layer situated on the Earth and having a thickness varying from day to night, namely from more than 1000 m to cca 10 m. Outside this boundary layer the geostrophic wind is about 10 ms^{-1}. In the troposphere the characteristic length in the horizontal direction is 10^6 m while in the vertical direction is 10^4 m. Since the coefficient of kinematic viscosity is $\nu \approx 1.1 \cdot 10^{-5}\ \mathrm{m}^2\mathrm{s}^{-1}$, it follows that the Reynolds number belongs to the interval $(4 \cdot 10^{16},\ 4 \cdot 10^{12})$, *i.e.*, the flow in the troposphere is turbulent. Since in the troposphere the influence of the temperature is very important, it follows that Eckman layer is a dynamic and thermal boundary layer. The mean motion in the troposphere is called the *general circulation*, while the random motion is characterized by coherent structures of various forms and dimensions. The upper layer of the atmosphere is referred to as the *stratosphere* and it is thermally influenced by the ozone. The layer situated between the stratosphere and the troposphere is called the *tropopause* and it is very thin with respect to the troposphere. In the horizontal directions in the atmosphere there are four characteristic dimensions (scales) which differ from the continuum scale and turbulence scale. They are the dimensions of the domain of motion Ω where specific types of flows occur.

The *planetary scale* has the characteristic length in the interval 4000 km $-$ 40000 km. The corresponding typical motions are: planetary waves, the jet, the maree, and they depend on seasons. The best suited model is the generalized Navier-Stokes model for transient flows. The *synoptical scale* has the characteristic length in the interval 1000 km $-$ 4000 km. The corresponding typical motions are coherent structures, which at moderate latitudes are the cyclones and anticyclones, while at the Equator are the hurricanes, Est waves. These motions are preturbulent and they are called the geostrophic turbulence. The planetary and synoptical scales are large scales. The *mesoscale* ranges in 10 km $-$ 1000 km correspond to motions presenting fronts, instability lines, relief forced flows and, possibly, turbulent flows. The *local scale* is smaller than 10 km and it is of interest for urban meteorology and pollution.

Meteorology is a domain of study which must take into account the complex features of the atmosphere fluid and its complicated motion in various domains and at various scales. It is mainly based on thermodynamics of fluids and uses several corresponding mathematical models to describe these turbulent fluid flows. We recall that in turbulence a lot of mathematical models were proposed, most of them including the (supported by experiments) *Reynolds idea* of splitting the fluid flow characteristics in *mean motion* and (random) *fluctuations*. This implies the subsequent use of closure relations connecting the Reynolds tensor to the mean flow characteristics to produce the so-called *naïve turbulence models*.

Apart from these models we quote the stochastic models, the Hopf type models and the Foiaş-Prodi statistical models (Foiaş 1972, 1973; Stanišić 1985). The last ones are related to the deterministic chaos, appropriate for the description of the preturbulent stage. Irrespective of the used model it contains several small parameters, asking for an *a priori* determination of their relative order of magnitude. Vamoş and Georgescu (1990) established these orders by using the results of the measurements for a long period of time. The relative order of one small parameter, which followed from some consistency relationships, remained to establish. Consequently, all small parameters were connected to the small parameter ε, namely the Rossby number. Then the method of regular asymptotic approximation was used to derive rigorously a model of the fifth asymptotic approximation, long ago deduced heuristically from the primitive equations, frequently used in meteorology (Holton 1979; Pedlosky 1987; Draghici 1988). Vamoş and Georgescu (1989a) deduced the same equations of asymptotic approximation, in the form of filtered equations, from the primitive equations by means of the method proposed by Vamoş and Georgescu (1990) but using the isobaric coordinates, namely φ, latitude, λ, longitude, p, pressure, instead of the isobar height $h(\varphi,\lambda,p,t)$. In other words, the filtered equations are the primitive equations in the so-called geostrophic approximation. Marinoschi and Georgescu (1992) took into account the orography to derive models of asymptotic approximation for the motion of the atmosphere over a low obstacle.

In Section **2**, we reveal the basic arguments and hypotheses used by Vamoş and Georgescu (1990) and in Section **3** those by Marinoschi and Georgescu (1992). We finally mention the crucial importance of the asymptotic methods in the studies of the atmosphere dynamics. Our results use these methods along the lines presented by Georgescu (1995).

2. Flows at synoptical scale

In this Section, we write the Euler model of the free atmosphere for flows at synoptical scale in coordinates appropriate to this situation. Then, following Vamoş and Georgescu (1989b), order relationships between four of the occurring small parameters are assumed such that only two small parameters remain in the equations. Further the method of regular perturbation is sketched.

First, instead of the spherical coordinates r, distance from the center of the Earth, θ, colatitude, and φ, longitude, let us introduce the new coordinates x, y, z where

$$x = \varphi\, a\cos\theta_0\ , \qquad y = (\theta-\theta_0)\, a\ , \qquad z = r-1\ ,$$

where a is a mean radius of the Earth and the constant θ_0 is the colatitude on which the domain of motion of interest is centered. For $x << a$ and $y << a$, then x and y approximate the distances measured at the surface of the Earth along the parallels and meridians, respectively.

Let $L,\ h,\ \tau = L/U,\ U,\ W,\ P,\ \mathscr{T}$ and D be characteristic quantities and we introduce the new non-dimensional quantities $x',\ y',\ z',\ t',\ \tau',\ u',\ v',\ w',\ p',\ \rho',$ with

$$\begin{aligned} &x = Lx'\ , \quad y = Ly'\ , \quad z = hz'\ , \quad t = \tau t'\ , \quad T = \mathscr{T}\tau'\ ,\\ &u = Uu'\ , \quad v = Uv'\ , \quad w = Ww'\ , \quad p = Pp'\ , \quad \rho = D\rho'\ , \end{aligned}$$

where $\mathbf{v} = (u,v,w)$, p, T and ρ are the velocity, pressure, temperature and density field, respectively. We assume $P = RD\mathscr{T}$ and $P = gDh$, *i.e.*, the characteristic quantities correspond to an ideal gas in hydrostatic equilibrium. This relation implies $gh = R\mathscr{T}$. Then, we assume the order of magnitude of the characteristic time is the same with the advective time, *i.e.*, $\tau = LU^{-1}$. In this way in the corresponding non-dimensional Euler model the following small physical parameters occur: the Rossby number $\varepsilon = (f_0\tau)^{-1} = U(f_0L)^{-1}$, the aspect ratio $\delta = h/L$, the aspect ratio characterizing the planeity of the domain of motion $\alpha = L/a$, a parameter characterizing the order of magnitude of the vertical velocity $\eta = \tau W/h$, and $\chi = f_0 U/g$, equal to the ratio between the Coriolis force and weight and characterizing the flow nonhydrostaticity; here $f_0 = 2\Omega\sin\theta_0$ and g is the gravitation acceleration.

We adopt the notation

$$\begin{aligned} A &= \cos(\theta_0+\alpha y)/\cos\theta_0, \quad B = \cos(\theta_0+\alpha y)/\sin\theta_0\,, \\ C &= \sin(\theta_0+\alpha y)/\sin\theta_0, \quad E = (\sin\theta_0+\alpha y)/\cos\theta_0\,, \end{aligned}$$

then the non-dimensional Euler model, where the primes are dropped, reads

$$\begin{aligned} &\rho\left[\chi\varepsilon\delta^{-1}\frac{du}{dt}+\chi\eta\varepsilon\alpha Auw-\chi\alpha\varepsilon\delta^{-1}Euv-\chi\delta^{-1}AC(1+\alpha\delta_z)v+\chi\eta AB(1+\alpha\delta_z)w\right] \\ &=-\frac{\partial p}{\partial x}\,, \\ &\rho\left[\chi\varepsilon\delta^{-1}\frac{dv}{dt}+\chi\eta\varepsilon\alpha Avw-\chi\alpha\varepsilon\delta^{-1}Eu^2+\chi\delta^{-1}AC(1+\alpha\delta_z)u\right]=-A\frac{\partial p}{\partial y}\,, \\ &\rho\left[\chi\varepsilon\eta\delta\frac{dw}{dt}-\chi\varepsilon\alpha A(u^2+v^2)-\chi AB(1+\alpha\delta_z)u+A(1+\alpha\delta_z)\right]=-A(1+\alpha\delta_z)\frac{\partial p}{\partial z}\,, \\ &\frac{dT}{dt}+\frac{R}{c_v}T\,div\mathbf{v}=0\,, \\ &\frac{d\rho}{dt}+\rho\,div\mathbf{v}=0\,, \\ &p=\rho T\,, \end{aligned} \tag{1}$$

where

$$\begin{aligned} div\mathbf{v} &= \eta A(1+\alpha\delta_z)\frac{\partial w}{\partial z}+2A\eta\alpha\delta w+A\frac{\partial v}{\partial y}+\alpha Ev+\frac{\partial u}{\partial x}\,, \\ \frac{d}{dt} &= A(1+\alpha\delta_z)\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+Av\frac{\partial}{\partial y}+\eta A(1+\alpha\delta_z)w\frac{\partial}{\partial z}\,. \end{aligned}$$

Assuming, as usual in studies on troposphere,

$$\begin{aligned} &U\approx 10\text{ m/s}\,,\quad L\approx 10^6\text{ m}\,,\quad h\approx 10^4\text{ m}\,,\quad P\approx 5\cdot 10^4\text{ N/m}^2\,,\quad \tau\approx 10^5\text{ s}\,, \\ &D\approx 0.5\text{ kg/m}^3\,,\qquad \mathscr{T}\approx 350\text{ K}\,,\qquad a\approx 6\cdot 10^6\text{ m}\,,\qquad f_0=10^{-4}\text{ s}\,, \end{aligned}$$

the non-dimensional parameters are

$$\varepsilon=10^{-1}\,,\qquad \delta=10^{-2}\,,\qquad \alpha=2\cdot 10^{-1}\,,\qquad \chi=10^{-2}\,,$$

therefore, assimilating 10^{-1} with an order of magnitude, we deduce the relationships between four of the five small parameters

$$\delta=O(\varepsilon^2)\,,\quad \alpha=O(\varepsilon)\,,\quad \chi=O(\varepsilon^4)\,,\qquad as\quad \varepsilon\to 0\,. \tag{2}$$

In this way we choose ε as the small parameter of the problem, of course after deducing the order relation between ε and η such that the equations (1) be asymptotically noncontradictory. Further, we apply the method of regular perturbations. Thus, we assume all unknown functions possess asymptotic expansions with respect to the asymptotic sequence $1,\varepsilon,\varepsilon^2,\cdots$ as $\varepsilon \to 0$, we introduce these expansions in (1) and deduce the models of asymptotic approximation of various orders $k = 0,1,\cdots$. However, as usual in asymptotics, some extra information is required in the course of computations. They are: $T_0 = 1$, $T_1(z)$ must be given (*i.e.*, the thermal stratification of the atmosphere must be specified *a priori*); $p_1\big|_z = 0$ must be specified too. Without this information, provided by observations, the method cannot go on. The consistency of the equations of the model of the zeroth order of asymptotic approximation implies

$$\eta = O(\varepsilon^2) \qquad \text{as} \qquad \varepsilon \to 0 \ .$$

Further on the reasoning are formal. Details are given by Vamoş and Georgescu (1990).

3. Atmospheric motion over a low obstacle

We consider the motion of the atmosphere near the Earth in the presence of a low obstacle. We assume the atmosphere is an inviscid compressible perfect gas and neglect the Earth rotation. The maximum height of the obstacle is $h_0 \sim 10^3$ m while the domain of motion has the eight (of the troposphere) $H \sim 10^4$ m. We denote by x, y and z the horizontal and vertical coordinates respectively, let $p_s = \rho_s R T_s$, ρ_s and T_s be the unperturbed pressure, density and temperature corresponding to the absence of the obstacle. We assume the motion is plane and it is characterized by the velocity $\mathbf{v} = (u,v,w)$, pressure p and temperature T and we take Ω as the domain of motion

$$\Omega = \{(x,y,z) \in \mathbb{R}^3 \mid (x,y) \in \Omega_z \ , \ z \in (h(x,y),H)\} \ ,$$
$$\Omega_z = \{(x,y) \in \mathbb{R}^2 \mid x \in [-L_x, L_x] \ , \ y \in [-L_y, L_y]\} \ ,$$

where $h(x,y)$ is the height of the atmosphere at the point (x,y) and L_x and L_y are sufficiently long to justify the vanishing of waves inside the domain Ω, such that:

$$\begin{aligned} &\text{as} \qquad x \to \pm L_x \, , \qquad y \to \pm L_y \, , \\ &\mathbf{v}(t,x,y,z) \to \mathbf{v}_s = (u_s, v_s, w_s) \ , \quad p(t,x,y,z) \to p_s \ , \quad T(t,x,y,z) \to T_s \ , \quad \rho \to \rho_s \ , \end{aligned}$$

where $w_s = h_0 u_s / l$, with l characteristic length, while as $z \to H$, $w = 0$ and on the obstacle $\mathbf{v} \cdot \mathbf{n} = 0$ on $z = h(x,y)$. Furthermore, u_s, v_s, w_s, p_s, T_s and ρ_s are unperturbed quantities.

We suppose that the unperturbed motion corresponds to the hydrostatic equilibrium, *i.e.*, $\frac{dp_s}{dz} = -\rho_0 g$, where $g = 10$ m/s^2 is the gravitation acceleration, $\rho_s(z) = \rho_s(0) e^{-\beta z}$, $T_s = T_s(z)$ (known, given) is the thermal stratification, $p_s(z) = \rho_s(z) R T_s(z)$, $R = R^* \mu_a^{-1}$, μ_a being the molar mass of air. We define the sound speed as $c_s^2(z) = \gamma p_s / \rho_s$, with $\gamma = 1,4$, which means that the undisturbed motion is adiabatic.

Let us pass to the non-dimensional quantities

$$\begin{aligned} &x' = x/l \ , \qquad y' = y/l \ , \qquad z' = z/H \ , \qquad t' = t u_s / l \ , \qquad \delta = h_0 / l \ , \\ &\mu = h_0 / H \ , \qquad \varepsilon = H / l \ , \qquad l_x = L_x / l \ , \qquad l_y = L_y / l \ , \end{aligned}$$

and $\mathscr{M}a = u_s/c_s$ (Mach number), $\mathscr{B}_0 = gH/c_s^2$ (Boussinesque number), βH (stratification exponent), and $\nu = 0.4$. Correspondingly, the domain Ω and the boundary conditions read:

$$\begin{aligned}
&\Omega = \{(x',y',z') \in \mathbb{R}^3 \mid x' \in (-l_x, l_x), y' \in (-l_y, l_y), z' \in (0,1)\} \ , \\
&\mathbf{v}' \to \mathbf{v}_s = (u_s, v_s, 0) \ , \quad p' \to p_s \ , \quad \rho' \to \rho_s \ , \quad T' \to T_s \ , \quad \text{as} \quad x' \to \pm l_x \ , \quad y' \to \pm l_y \ , \\
&w' \to 0 \quad \text{as} \quad z' \to 1 \ , \\
&u' \frac{\partial \overline{h}}{\partial x'} + v' \frac{\partial \overline{h}}{\partial y'} = w' \quad \text{as} \quad z \to 0 \ ,
\end{aligned}$$

where $h(x,y) = h_0 \overline{h}(x'l, y'l)$ and, in the hypothesis that the atmospheric boundary is small, the condition on the obstacle surface was replaced by the condition on the Earth without obstacle, *i.e.*, at $z' = 0$.

Finally, we introduce the new unknown functions

$$\mathbf{V} = (\rho_0')^{1/2} \mathbf{v}' \ , \qquad P = (\rho_0')^{-1/2} p'$$

and drop the primes. Then the governing model, consisting in the constitutive equation, Euler equations and the boundary conditions (since at this stage the initial conditions are not of interest), reads in the following form appropriate to an asymptotic analysis:

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \rho_0^{-1/2} \rho \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \mu \frac{\partial W}{\partial z} \right) + \rho_0^{-1/2} \rho \left(U \frac{\partial \rho}{\partial x} + V \frac{\partial \rho}{\partial y} + \mu W \frac{\partial \rho}{\partial z} \right) \\
&- \mu \frac{1}{2\rho_0} \frac{d\rho_0}{dz} \rho_0^{-1/2} \rho W = 0 \ , \\
&\mathscr{M}a^2 \gamma \left[\rho \frac{\partial U}{\partial t} + \rho_0^{-1/2} \rho \left(U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + \mu W \frac{\partial U}{\partial z} \right) - \mu \frac{1}{2\rho_0} \frac{d\rho_0}{dz} \rho_0^{-1/2} \rho W U \right] \\
&= - \frac{\partial P}{\partial x} \rho_0 \ , \\
&\mathscr{M}a^2 \gamma \left[\rho \frac{\partial V}{\partial t} + \rho_0^{-1/2} \rho \left(U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + \mu W \frac{\partial V}{\partial z} \right) - \mu \frac{1}{2\rho_0} \frac{d\rho_0}{dz} \rho_0^{-1/2} \rho W V \right] \\
&= - \frac{\partial P}{\partial y} \rho_0 \ , \\
&\mathscr{M}a^2 \gamma \varepsilon^2 \mu \left[\rho \frac{\partial W}{\partial t} + \rho_0^{-1/2} \rho \left(U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + \mu W \frac{\partial W}{\partial z} \right) - \mu \frac{1}{2\rho_0} \frac{d\rho_0}{dz} \rho_0^{-1/2} \rho W W \right] \\
&= - \frac{\partial P}{\partial z} \rho_0 - \mathscr{B}_0 \rho_0^{1/2} \rho - \frac{1}{2} \frac{d\rho_0}{dz} P \ , \\
&P = \rho_0^{-1/2} \rho T \ , \\
&\rho_0^{1/2} \frac{\partial T}{\partial t} + \left(U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} + \mu W \frac{\partial T}{\partial z} \right) + \nu T \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \mu \frac{\partial W}{\partial z} \right) \\
&- \mu \nu \frac{1}{2\rho_0} \frac{d\rho_0}{dz} T W = 0 \ ,
\end{aligned} \tag{3}$$

$$\begin{aligned}&\mathbf{V} \to \mathbf{V_0}\,, \qquad P \to P_0\,, \qquad \rho \to \rho_0\,, \quad \text{as} \qquad x \to \pm l_x\,, \qquad y \to \pm l_y\,,\\ &W \to 0 \qquad \text{as} \qquad z \to 1 \qquad U\frac{\partial \overline{h}}{\partial x} + V\frac{\partial \overline{h}}{\partial y} = W \quad \text{as} \quad z \to 0\,.\end{aligned} \tag{4}$$

Choosing, as customary in troposphere,

$$l = H \sim 10^4 \text{ m}\,, \qquad c_s \sim 340 \text{ m/s}\,, \qquad \beta = 10^{-4} \text{ m}\,,$$

and taking $\mu = 10^{-1}$ as the small parameter of the problem, we have

$$\begin{aligned}&\mu = 10^{-1}\,, \qquad \delta = 10^{-1} = O(1)\,, \qquad \mathscr{M}_a^2 = 10^{-3} = O(\mu^3)\,, \qquad \varepsilon = 1\,,\\ &\nu = \sigma(\mu)\,, \qquad \beta H = O(1)\,, \qquad \varepsilon = O(1)\,, \qquad \mathscr{B}_0 = 0(1) \qquad \text{as} \qquad \mu \to 0\,.\end{aligned}$$

Finally, choosing as the zeroth approximation the unperturbed motion, we further can apply the regular perturbation method to equations (3) and (4) to derive the models of asymptotic approximation of higher orders. Detailed computations were performed by Marinoschi and Georgescu (1992).

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Editor's note

Prof. Adelina Georgescu, a former corresponding member of the "Accademia Peloritana dei Pericolanti", passed away on May 1st, 2010.[†] *She delivered this lecture on the occasion of a meeting entitled "Modeling in Meteorology", which was held at the University of Messina in 2007. Prof. Liliana Restuccia, a close collaborator of the author, has taken care of the review and editing of the draft manuscript that Prof. Adelina Georgescu had originally prepared for the meeting and that is published posthumously in grateful memory of a highly respected, profoundly generous colleague.*

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