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THE INTERTEMPORAL CHOICE BEHAVIOUR
THE ROLE OF EMOTIONS IN A MULTI-AGENT DECISION PROBLEM

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ABSTRACT. Decision Neuroscience has shown positive and negative side of emotions in intertemporal choices. Psychological evidences, indeed, point out anomalies (impulsivity modifies the discount function of each individual) and the false consensus effect which increases the degree of consensus in a multi-agent decision problem. An experiment (Engelmann and Strobel 2004) demonstrates that the relevance of the false consensus effect depends on the difficulty of the information retrieval, so the underlying mechanism is an information processing deficiency rather than egocentricity. We demonstrate that emotions can not cause anomalies in a cooperative strategic interaction because information is explicit.

1. Introduction

Traditional Discounted Utility Model (DU model) assumes an exponential delay discount function, with a constant discount rate that implies dynamic consistency and stationary intertemporal preferences. Contrary to this normative economic theory, decision neuroscience has established that human and animal intertemporal choice behaviors are not rational (i.e., inconsistent) (Sec. **2**). The DU model fails in being both normative and descriptive, as shown by several studies especially carried out in the fields of psychology and neuroeconomics, that reveal the existence of relevant anomalies, which violate the traditional model axioms (Sec. **3**).

Bechara *et al.* (1997) show that decision making processes are considerably guided by emotional signaling, which allow people to choose advantageously before they may realize which strategy works best. This fact justifies the presence of anomalies in intertemporal choice and the use of hyperbolic delay discounting (declining as the length of the delay increases), so, people have the tendency to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time. This entails intertemporal inconsistency and preferences reversal. Even so, an impatient behavior not necessarily is incoherent. (Sec. **4**)

The results of a study of Shiv *et al.* (2005) demonstrated that patients with lesions in specific components of a neural circuitry critical for the processing of emotions will make more advantageous decisions than normal subjects when faced with the types of positive-expected-value gambles that most people routinely shun (Sec. **5**). Recent neuroeconomic and econophysical studies have explored neurobiological and psychological factors (as impulsivity and inconsistency) which determined individual differences in intertemporal choice. Econophysical studies have proposed the Q-exponential Delay Discount Function to dissociate impulsivity and inconsistency. Other behavioral economists have proposed Multiple Selves Models attempting to measure the strength of the internal conflict within the decision maker, best known as quasi-hyperbolic discount model first introduced by Laibson in 1997 (Sec. **6**).

To fight impulsivity Strotz (1955–1956) proposed two strategies that might be employed by a person who foresees how her preferences will change over time, and Thaler and Shefrin (1981) proposed a model in which individual is treated as if he contained two distinct psyches denoted as planner and doer (Sec. **7**).

In a multi-agent decision context the objective for a decision group is to choose a common decision, that is an alternative which is judged the best by the majority of the decision makers. So in most strategic decisions, it is important to be able to estimate the characteristics and behavior of others. If the characteristics of other players are unknown, estimating them is a critical task (Sec. **8**). Moreover, psychological evidence suggests people's own beliefs, values, and habits tend to bias their perceptions of how widely they are shared (false consensus effect). This effect demonstrates an inability of individuals to process information rationally (Sec. **9**). Therefore, when we use the aggregation of the agents' preferences to assess consensus, we obtain a coefficient which includes the false consensus effect that depends on the subjectivity and also increases the degree of consensus. To eliminate this aspect of human judgment vagueness we can use a model defined by ordered weighted averaging (OWA) operators (Sec. **10**).

An experiment of Engelmann and Strobel (2004) demonstrates that a false consensus effect is present only if information about decision of other members of the group is implicit. So the consensus effect is not always false but only when people, forming expectations concerning other people's decisions, weight their own decision more heavily than that of a randomly selected person from the same population (Sec. **11**). This result is linked with analysis of false consensus effect in cooperative and non-cooperative decision problem. Indeed in cooperative decision problem agents know choices of other members, while in non-cooperative one they have to judge others choices (Sec. **12**).

2. Traditional discounting model and decision neuroscience

The standard economic model of discounted utility (DU model) assumes that economic agents make intertemporal choices over consumption profiles $(c_t,\ldots,c_T)$ and such preferences can be represented by an intertemporal utility function $U^t(c_t,\ldots,c_T)$, which can be described by the following special functional form:

$$U^t(c_t,\ldots,c_T) = \sum_{k=0}^{T-t} D(k)u(c_{t+k}) \qquad \text{where} \quad D(k) = \left(\frac{1}{1+\rho}\right)^k .$$

Hence, the DU model assumes an exponential temporal discounting function and a constant discount rate (ρ), which represents the individual's pure rate of time preference.

An important implication of constant discount rate and exponential discounting function is that a person's intertemporal preferences are time-consistent: if in period t a person prefers c_2 at $t+2$ to c_1 at $t+1$, then in period $t+1$ she must prefer c_2 at $t+2$ to c_1 instantly. So, with the same temporal options and the same information, later preferences confirm earlier preferences. However, several empirical studies have documented various inadequacies of the DU model as a descriptive model of behavior. Behavioral economic theories on decision process have found that there are a number of behavior patterns that violate the rational choice theory.

Decision neuroscience is an emerging area of research whose goal is to integrate research in neuroscience and behavioral decision-making. It has call into question the theories of choice which assume that decisions derive from an assessment of the future outcomes of various options and alternatives through some type of cost-benefit analyses, which ignore influence of emotions on decision-making. This field of study explores the neural "road map" for the physiological processes intervening between knowledge and behavior, and the potential interruptions that lead to a disconnection between what one knows and what one decides to do. The studies of decision-making in neurological patients who can no longer process emotional information normally suggest that people make judgments not only by evaluating the consequences and their probability of occurring, but also and even sometimes primarily at a gut or emotional level (Bechara 2004).

3. Behavioral finance: empirical anomalies violating the traditional discounting model

According psychology studies individual behavior, when discounting real or hypothetical rewards, showing the existence of violations of the traditional discounting models. First, empirically observed discount rates are not constant over time, but appear to decline - a pattern often referred to as hyperbolic discounting. Furthermore, even for a given delay, discount rates vary across different types of intertemporal choices (A. G. S. Ventre and V. Ventre 2012). Delay effect, magnitude effect, sign effect and sequence effect are among the relevant anomalies in intertemporal choice, we shall deal with.

The delay effect. As waiting time increases, the discount rates tend to be higher in short intervals than in longer ones. This anomaly is also called common difference effect and immediacy effect. We can set out this effect as follows:

$$(x,s) \sim (y,t) \quad \text{but} \quad (x,s+h) < (y,t+h), \quad \text{for} \quad y > x,\, s < t \quad \text{and} \quad h > 0\,.$$

If two capitals, (x,s) and (y,t), are indifferent, $(x,s) \sim (y,t)$, their projections onto a common instant p (usually, p is taken as 0) have to coincide:

$$xA(s,p) = yA(t,p) \quad \text{if and only if} \quad \frac{x}{y} = \frac{A(t,p)}{A(s,p)} = v(s,t,p)\,,$$

$A(t,p)$ being the discount function which represents the amount available at p instead of one euro available at t, and $v(s,t,p)$ the corresponding financial factor. In the same way, if $(x,s+h)(y,t+h)$, this implies that

$$xA(s+h,p) = yA(t+h,p) \quad \text{if and only if} \quad \frac{x}{y} = \frac{A(t+h,p)}{A(s+h,p)} = v(s+h,t+h,p)\,.$$

Then

$$v(s,t,p) < v(s+h,t+h,p).$$

The magnitude effect. Larger outcomes are discounted at a lower rate than smaller outcomes. Let us suppose that the instantaneous discount rate is inversely proportional to the discounted amount:

$$\delta(z) = \frac{k}{c'} \quad \text{with} \quad k > 100.$$

It follows that

$$A(c,z) = c \cdot e^{-\int_0^z \frac{k}{c}\, dx} = c \cdot e^{-\frac{k}{c} z}.$$

This effect can be formulated as follows:

$$(x,s) \sim (y,t) \quad \text{implies} \quad (\alpha x,s) < (\alpha y,t), \quad \text{for} \quad y > x > 0, \quad s < t\,,$$

and

$$(-x,s) \sim (-y,t) \quad \text{implies} \quad (-\alpha x,s) > (-\alpha y,t).$$

The sign effect. Gains are discounted at a higher rate than losses of the same magnitude. This anomaly is also called "amplification loss property" implying that, changing the sign of an amount from gains to losses, the weight of this amount increases:

$$(x,s) \sim (y,t) \quad \text{implies} \quad (-x,s) > (-y,t) \quad \text{for} \quad y > x > 0 \quad \text{and} \quad s < t.$$

The improving sequence effect. Increasing sequences of consumption are preferred over decreasing ones even if the total amount is the same. In general, when subjects choose among different sequences of two events people tend to save the better thing for last, contradicting the standard assumption of a positive interest rate. The improving sequence effect can be characterized as follows: for all s and t, and $s < t$, there is a c_0 such that, for all $y > x > c_0$, the following relation holds (A. G. S. Ventre and V. Ventre 2012):

$$\{(x,s),(y,t)\} >\ _p \{(y,s),(x,t)\}.$$

4. Anticipation of future events and hyperbolic discounting

In contrast to the historically dominant view of emotions as a negative influence in human behavior, recent research in neuroscience and psychology has highlighted the positive roles played by emotions in decision making (Bechara *et al.* 1997; Damasio 1994). Notwithstanding the fact that strong negative emotions such as jealousy and anger can lead to destructive patterns of behavior such as crimes of passion and road rage, in a series of studies using a gambling task, researchers have shown that individuals with emotional dysfunction tend to perform poorly compared with those who have intact emotional processes (Bechara *et al.* 1997; Damasio 1994; Shiv *et al.* 2005).

Bechara *et al.* (1997) concluded with an experiment that decision making is guided by emotional signaling generated in anticipation of future events. Without the ability to generate these emotional signals, the patients fail to avoid choices that lead to losses, and instead continue to sample from the disadvantageous choices until they go broke in a manner that is akin to how they behave in real life. In normal individuals, nonconscious biases guide behavior before conscious knowledge does. Without the help of such biases, overt knowledge may be insufficient to ensure advantageous behavior (Bechara *et al.* 1997).

Decision-maker's preferences are inconsistent and change over time, because normal people possess anticipatory SCRs (indices of somatic states), which represent unconscious biases that are linked to prior experiences with reward and punishment. These biases alarm the normal subject about selecting a disadvantageous course of action, even before the subject becomes aware of the goodness or badness of the choice he is about to make. Indeed, when normal people won or lost money on an investment round, they adopted a conservative strategy and became more reluctant to invest on the subsequent round (Shiv *et al.* 2005). Furthermore the preference for more immediate rewards per se is not always irrational, because there are opportunity costs and risk associated with non-gaining in delaying the rewards. As consequence, there is considerable agreement among psychologists and economists that the notion of exponential discounting should be replaced by some form of hyperbolic discounting, which can represent the tendency of the individuals to increasingly choose a smaller-sooner reward over a larger-later reward as the delay occurs sooner in time (delay effect).

Many authors proposed different hyperbolic discount functions, in which δ (temporal discount function) increases with the delay to an outcome. One of these proposed functions has the form

$$d(t) = \left(\frac{1}{1+\alpha t}\right)^{\frac{\beta}{\alpha}},$$

where $\beta > 0$ is the degree of discounting and $\alpha > 0$ is the departure from exponential discounting. Hyperbolic discounting has been applied to a wide range of phenomena, including consumption-saving behavior. And, consistently with hyperbolic discounting, people's investment behavior exhibits patience in the long run and impatience in the short run (A. G. S. Ventre and V. Ventre 2012).

A second type of empirical support for hyperbolic discounting comes from experiments on dynamic inconsistency. Studies and empirical evidences show that delay effect can derive in preference reversal between two rewards as the time-distance to these rewards diminishes. A hyperbolic discount model can demonstrate this; in fact, non-exponential time-preference curves can cross (Strotz 1955–1956) and consequently the preference for one future reward over another may change with time (A. G. S. Ventre and V. Ventre 2012).

5. The negative side of emotions: impulsivity

The positive roles played by emotions in decision making contrast with some contexts in which individuals deprived of normal emotional reactions might actually make better decisions than normal individuals (Damasio 1994). For example, let us consider the case of a patient with ventromedial prefrontal damage (which involves severe impairments in judgment and emotion) who was driving under hazardous road conditions (Damasio 1994). When other drivers reached an icy patch, they hit their brakes in panic, causing their vehicles to skid out of control, but the patient crossed the icy patch unperturbed, gently pulling away from a tailspin and driving ahead safely. The patient remembered the fact that not hitting the brakes was the appropriate behavior, and his lack of fear allowed him to perform optimally (Shiv *et al.* 2005).

Other evidences suggest that even relatively mild negative emotions that do not result in a loss of self-control can play a counterproductive role among normal individuals in some situations. When gambles that involve some possible loss are presented one at a time, most people display extreme levels of risk aversion toward the gambles, a condition known as *myopic loss aversion*. If myopic loss aversion does indeed have an emotional basis, then any dysfunction in neural systems subserving emotion ought to result in reduced levels of risk aversion and, thus, lead to more advantageous decisions in cases in which risk taking is rewarded (Shiv *et al.* 2005). In this latter study the authors demonstrated these empirical evidences: individuals deprived of normal emotional reactions might, in certain situations, make more advantageous decisions than those not deprived of such reactions; so the lack of emotional reactions may lead to more advantageous decisions.

Indeed, in many cases temptations induce disadvantageous behavior, and when temptation becomes too great what the person knows to be his best long run interests conflict with his short run desires. Sociologists and psychologists have persistently studied impulsivity relative to its resultant behaviors such as drug addiction, suicide, aggression and violence. These studies suggests that individuals who frequently engage in impulsive behavior may fail to evaluate the consequences of their behavior appropriately (A. G. S. Ventre and V. Ventre 2012).

6. Neuroeconomics: impulsivity and inconsistency in intertemporal choice

In an intertemporal choice the greatest contradiction to rational theory is inconsistent preference, usually manifested as temporary preference for options that are extremely costly or harmful in the long run. This behavior can be typically seen in psychiatric disorders (alcoholism, drug abuse), but also in more ordinary phenomena (overeating, credit card debt) (A. G. S. Ventre and V. Ventre 2012). Neuroeconomics (a more specialized field of decision neuroscience) has found that addicts are more myopic (have large time-discount rates) in comparison to non-addicted populations, so hyperbolic discounting may explain various human problematic behaviors: loss of self-control, failure in planned abstinence from addictive substances and relapse, a dead-line rush due to procrastination, failure in saving enough before retirement and risky sexual behavior. Addiction and financial mismanagement frequently co-occur, and elevated delay discounting may be a common mechanism contributing to both of these problematic behaviors.

We have noted that the preference for more immediate rewards per se is not always irrational or inconsistent (Sec. **4**); therefore, impulsivity in intertemporal choice is rationalizable for several kinds of people. Addicts' behavior is clinically problematic, but economically rational when their choices are time-consistent (if they have large discount rates with an exponential discount function). However, it is known that addicts also discount delayed outcomes hyperbolically, suggesting the intertemporal choices of addicts are time-inconsistent, resulting in a loss of self-control: they act more impulsively at the moment of the choice, against their own previously-intended plan. Moreover if large discount rates are due to habitual drug intake, it is expected that discount rates decreased after long-term abstinence. However, recent studies have reported that for alcoholics and smokers, abstinence did not dramatically reduce discount rates of former alcoholics and smokers. Recently, behavioral neuroeconomic and econophysical studies have proposed two discount models, in order to better describe the neural and behavioral correlates of impulsivity and inconsistency in intertemporal choice.

Q-exponential discount model. This function has been proposed and examined for subjective value V(D) of delayed reward:

$$V(D) = \frac{A}{exp_q(k_q D)} = A/[1+(1-q)k_q D]^{\frac{1}{1-q}},$$

where D denotes a delay until receipt of a reward, A the value of a reward at $D = 0$, and k_q a parameter of impulsivity at delay $D = 0$ (q-exponential discount rate) and the q-exponential function is defined as:

$$exp_q(x) = (1+(1-q))^{\frac{1}{1-q}}.$$

This function can distinctly parametrize impulsivity and inconsistency (A. G. S. Ventre and V. Ventre 2012).

Quasi-hyperbolic discount model. Behavioral economists have proposed that the inconsistency in intertemporal choice may be attributable to an internal conflict between "multiple selves" within a decision maker. As a consequence, there are (at least) two exponential discounting selves (with two exponential discount rates) in a single human individual; and when delayed rewards are at the distant future (> 1 year), the self with a smaller discount rate wins, while delayed rewards approach to the near future (within a year), the self with a larger discount rate wins, resulting in preference reversal over time. This intertemporal choice behavior can be parametrized in a quasi-hyperbolic discount model (also as a $\beta - \delta$ model). For a discrete time τ (the assumed unit is one year), it is defined as:

$$F(\tau) = \beta\delta^t \quad (\text{for } \tau = 1,2,3,\dots) \quad \text{and} \quad F(0) = 1 \quad (0 < \beta < \delta < 1).$$

A discount factor between the present and one-time period later (β) is smaller than that between two future time-periods (δ). In the continuous time, the proposed model is equivalent to the linearly-weighted two-exponential functions (generalized quasi-hyperbolic discounting):

$$V(D) = A[w\,exp(-k_1 D) + (1-w)\,exp(-k_2 D)],$$

where w, $0 < w < 1$, is a weighting parameter and k_1 and k_2 are two exponential discount rates ($k_1 < k_2$). Note that the larger exponential discount rate of the two k_2, corresponds to an impulsive self, while the smaller discount rate k_1 corresponds to a patient self (A. G. S. Ventre and V. Ventre 2012).

7. Self-control against impulsivity: Strotz and Thaler's and Shefrin's models

A number of mechanisms of self-control are predicted by hyperbolic discounting. Strotz (1955–1956) proposed two strategies that might be employed by a person who foresees how her preferences will change over time:

(1) *The "strategy of precommitment"*: a person can commits to some plan of action. For example, consider a consumer with an initial endowment K_0 of consumer goods which has to be allocated over the finite interval $(0, T)$. At time period t he wishes to maximize his utility function:

$$J_0 = \int_0^T \lambda(t-0)U[\bar{c}(t),t]dt \quad \text{subject to} \quad \int_0^T c(t)dt = K_0\,,$$

where $[\bar{c}(t),t]$, is the instantaneous rate of consumption at time period t, and $\lambda(t-0)$ is a discount factor, the value of which depends upon the elapse of time between a past or future date and present. And this implies that the discounted marginal utility of consumption should be the same for all periods. But, at a later date, the consumer may reconsider his consumption plan. The problem then is to maximize

$$J_0 = \int_0^T \lambda(t-\tau)U[c(t),t]dt \quad \text{subject to} \quad \int_\tau^T c(t)dt = K_\tau = K_0 - \int_0^\tau c(t)dt\,.$$

The optimal pattern of consumption will change with changes in τ and if the original plan is altered, the individual is said to display dynamic inconsistency. Strotz showed that individuals will not alter the original plan only if $\lambda(t,\tau)$ is exponential in $|t-\tau|$.

(2) *The "strategy of consistent planning"*: since precommitment is not always a feasible solution to the problem of intertemporal conflict, an individual may adopt a different strategy: take into account future changes in the utility function and reject any plan that he will not follow through. His problem is then to find the best plan among those he will actually follow.

In the setting of Multiple Selves Models, Thaler and Shefrin (1981) proposed a "planner-doer" model to control impulsivity which draws upon principal-agent theory. They treat an individual as if he contained two distinct psyches: one *planner*, which pursue longer-run results, and multiple *doers*, which are concerned only with short-term satisfactions, so they care only about their own immediate gratification (and have no affinity for future or past doers). For example, consider an individual with a fixed income stream $y = [y_1, y_2, \ldots, y_T]$, where

$$\sum_t y_t = Y\,,$$

which has to be allocated over the finite interval $(0, T)$. The planner would choose a consumption plan to maximize his utility function

$$V(Z_1, Z_2, \ldots, Z_T) \quad \text{subject to} \quad \sum_{t=1}^{t} c_t \le Y\,,$$

in which such Z_t is a function of utility of level consumption in t (c_t).

On the other hand, an unrestrained doer 1 would borrow $Y - y_1$ on the capital market and therefore choose $c_1 = Y$; the resulting consequence is naturally $c_2 = c_3 = \ldots = c_T = 0$. Such action would suggest a complete absence of psychic integration. Then the model focuses on the strategies employed by the planner to control the behavior of the doers, and it proposes two instruments he can use. (a) He can impose *rules* on the doers' behavior, which operate by altering the constraints imposed on any given doer; or (b) he can use *discretion* accompanied by some method of altering the incentives or rewards to the doer without any self-imposed constraints (A. G. S. Ventre and V. Ventre 2012).

8. Multi-agent decision problem: consensus and agreement

In a multi-agent decision problem an individual needs to take his intertemporal choice considering others' preferences, to the purpose of achieving a consensus on a common decision. Group decision problems, indeed, consist in finding the best alternative(s) from a set of feasible alternatives $A = \{a_1, \ldots, a_n\}$ according to the preferences provided by a group of agents $E = \{e_1, \ldots, e_m\}$. The objective is to obtain the maximum degree of agreement among the agents' overall performance judgments on the alternatives (Squillante and V. Ventre 2010). Once the alternatives have been evaluated, the main problem is to compare agents' judgments to verify the consensus among them; in the case of unanimous consensus, the evaluation process ends with the selection of the best alternative(s). However, in real situations humans rarely come to a unanimous agreement: this has led to evaluate not only crisp degrees of consensus (degree 1 for fully and unanimous agreement) but also intermediate degrees between 0 and 1 corresponding to partial agreement among all agents. Furthermore, full consensus (degree = 1) can be considered not necessarily as a result of unanimous agreement, but it can be obtained ever in the case of agreement among a fuzzy majority of agents.

9. False consensus

Researches in areas of social judgement have revealed that people are egocentric: they judge others in the same way that they judge themselves. Consequently, as pointed out in several experiments, in a multi-agent decision problem each decision maker overestimates his own opinion. Social psychology has founded that people with a certain preference tend to make higher judgments of the popularity of that preference in others, compared to the judgments of those with different preferences. This empirical result has been termed the *false consensus effect*. It states that individuals overestimate the number of the people who possess the same attributes as they do. People often believe that others are more like themselves than they really are. Thus, their predictions about others' beliefs or behaviors, based on casual observation, are very likely to err in the direction of their own beliefs or behavior. For example, college students who preferred brown bread estimated that over 50% of all other college students preferred brown bread, while white-bread eaters estimated that 37% showed brown bread preference. As a consequence, in multi-agent decision problems we often have to deal with different opinions, different importance of criteria and agents, who are not fully impartial objective. In this sense, the false consensus effect produces partial objectivity and incomplete impartiality, which perturbs the agreements over the evaluation (Squillante and V. Ventre 2010).

10. Assessing consensus and false consensus

In the literature, different methods to compute a degree of a consensus in fuzzy environments have been defined, and some approaches have been proposed to measure consensus in the context of fuzzy preference relations. However, as we have seen, the false consensus effect can lead to an absence of objectivity in the evaluation process. Then just a numerical indication seems not to be sufficient to synthesize the degree of consensus of agents which incorporate both the true knowledge generated agent opinion and the subjective component that produces false consensus outputs. The opinion of each agent is decomposed into two components: a vector, made of the ranking of the alternatives, built by means of a classical procedure, e.g., a hierarchical procedure, and a fuzzy component that represents the contribution of the false consensus effect, which we assume to be fuzzy in nature. This allows us to consider aggregation operators, such as OWA operators, useful when synthesis among fuzzy variables is to be built (Squillante and V. Ventre 2010).

The formal model considers the set N of decision makers, the set A of the alternatives, and the set C of the criteria. Let any decision maker $I \in N$ be able to assess the relevance of each criterion. Precisely, for every i, a function

$$h_i : C \to [0,1] \quad \text{with} \quad \sum_{c\in C} h_i(c) = 1$$

denoting the evaluation or weight that the decision maker assigns to the criterion c, is defined. Furthermore, the function

$$g_i : A \times C \to [0,1]$$

is defined, such that $g_i(a,c)$ is the value of the alternative a with respect to the criterion c, in the perspective of i.

Let n, p, and m denote the (positive integer) numbers of the elements of the sets N, C, and A, respectively. The value $h_i(c)_{c\in C}$ denotes the evaluation of the p-tuple of the criteria by the decision maker i and the value $g_i(c,a)_{c\in C, a\in A}$ denotes the matrix $p\times m$ whose elements are the evaluations, made by i, of the alternatives with respect to each criterion in C. Function: $A \to [0,1]$, defined by

$$(f_i(a))_{a\in A} = h_i(c)_{c\in C} \cdot g_i(c,a)_{c\in C, a\in A}\,,$$

is the evaluation, made by i, of the alternative $a \in A$.

An euclidean metric that acts between couples of decision makers i and j, i.e., between individual rankings of alternatives, is defined by

$$d(f_i, f_j) = \sqrt{\frac{1}{|A|}\sum_{a\in A}(f_i(a) - f_j(a))^2}\,.$$

If the functions h_i, g_i range in $[0,1]$, then also $0 \le d(f_i, f_j) \le 1$. If we set

$$d^* = \max\left\{d(f_i, f_j) | i, j \in N\right\},$$

then a degree of consensus δ^* can be defined as the complement to one of the maximum distance between two positions of the agents:

$$\delta^* = 1 - d^* = 1 - \max\left\{d(f_i, f_j) | i, j \in N\right\}.$$

Now, to identify the portion of the false consensus effect internal to the consensus-reaching process we have to consider a vector that represents the

$$\textit{components of the consensus} = p(a)P + q(a)Q.$$

This polynomial representation of the measure of the effect is composed by a numeric component $p(a)P$ that contains all quantitative information available derived from the consensus-reaching process, and $q(a)Q$ that reflects the false consensus effect. Then the measure of the effect is:

$$q(a) = \frac{1}{N(d^*)^2} \sum_{i=1}^{N} (f_i - f_j)^2 ,$$

$$\text{with} \quad 0 \leq q(a) \leq 1, \forall i, j \in N.$$

This component can be estimate with ordered weighted averaging (OWA) operators (belonging to a large class of decision support tools for providing heuristic solution to situations where several trade-offs should be taken into consideration). On OWA operators is based an approach for multiple criteria aggregation: by ranking the alternatives, the operators provide an enhanced methodology for evaluating actions on a qualitative basis (Squillante and V. Ventre 2010).

11. Study on false consensus effect under varying information conditions: Engelmann and Strobel's experiment

Most studies define false consensus as an egocentric bias that occurs when people estimate consensus for their own behaviors. The judgments of each agent, indeed, are frequently based, in part, on intuition or subjective beliefs, rather than detailed data on the preferences of the people being predicted. However such intuitive judgments become more pervasive judgments when people lack necessary data to base their judgments (Squillante and V. Ventre 2010). Therefore, classical definition of false consensus does not justify the label “false”. He argues that it is perfectly rational to use the information about one's own decision in the same way as the information about any other randomly selected sample of size one. The effect is only false if too much weight is assigned to one's own decision compared to a randomly selected person from the same population. Engelmann and Strobel (2004) refer to the effect as defined above as a consensus effect and affirms that people exhibit a false consensus effect if among those with the same total information (i.e. that includes the information about their own decision) the estimates are biased in the direction of their own decision. To demonstrate this and investigate whether a false consensus effect depends on the cognitive effort needed to retrieve information, Engelmann and Strobel compared two treatments in a simple one-shot experiment (Engelmann and Strobel 2004).

Results are in opposite direction to a false consensus effect when in a decision group the agents have explicit information about the choice of other members of their own group, while results are in line with a false consensus effect in all groups in which the information were implicit. This shows that most subjects are unwilling or unable to use information that is not handed to them on a silver platter. It appears to us that in the implicit information treatment it does not occur to many subjects that the other subjects' choices are valuable information and that this information is rather easily available, while the prominent information in the explicit information treatments is recognized as valuable information by virtually all subjects (or leads them to unconsciously update their beliefs).

In conclusion, Engelmann and Strobel can affirm that there is no false consensus effect if representative information is highly prominent and retrievable without any effort. Indeed, there is even a significant effect in the opposite direction, indicating that subjects consider others' choices as more informative than their own (Engelmann and Strobel 2004).

12. False consensus effect and emotions in a multi-agent decision problem

The result of Engelmann and Strobel's experiment on false consensus effect demonstrates that in a non-cooperative decision problem is only a chance obtaining a common decision. Multi-agent decision problems are characterized by interplay between intertemporal considerations and strategic interactions: two or more agents could have to take a common decision for a future time and in this process they are influenced by emotional signal, which arise with impulsivity and with "false" or "true" consensus effect. Mathematical instrument used to describe strategic interactions is the theory of games. Then, using theory of games and Thaler and Shefrin's model, we can demonstrate the difference of emotion influence between a cooperative interaction and non-cooperative one. This difference is linked with the result of Engelmann and Strobel's experiment.

In non-cooperative interaction, indeed, each agent makes decisions independently, without collaboration or communication with any of the others. This can be assimilated to situations in which information about decision of other members of decision group is implicit. In this kind of strategic decision the consensus effect is false. As in Engelmann and Strobel's experiment, indeed, if members of group decision do not cooperate they do not have information about others choices, so the influence of psychological aspects lead to judge others in the same way that they judge themselves. Then two situations are possible: 1) each agent have the same preference and they will reach a common decision that is given by the unanimous choice; 2) the agents have different preferences and do not assign any weight to the other preferences, so it is not possible to aggregate them (according to Sec. **10**).

Then, the influence of emotions do not have negative consequences if agents' choices are unanimous, and in this case the final decision will be also the best decision in term of Pareto, but if this does not happen is impossible to achieve a common strategy without arresting impulsivity, and unanimity becomes increasingly difficult to obtain when the number of agents increases. On the contrary, in a cooperative decision problem the influence of false consensus effect is present at period-one, while the loss of self-control of each agent is fought by the imposition of a rule (Thaler and Shefrin 1981). The rationality of the equilibrium choice of the cooperative game is saved by the possibility of making an arrangement among agents, which represents a pure rule to maintain self-control at later time in Thaler and Shefrin's model (Sec. **7**). Moreover with an arrangement the agents have explicit information about the choices of other members, so the lack of false consensus effect is in line with the result of Engelmann and Strobel's experiment.

Consider the classic example of coordination game: the "battle-of-the sexes". In this game an engaged couple must choose what to do in the evening: the man prefers to attend a baseball game and the women prefers to attend an opera. In term of utility the payoff for each strategy is:

		Man	
		Opera (O)	Baseball (B)
Woman	Opera (O)	3, 1	0, 0
	Baseball (B)	0, 0	1, 3

In this example there are multiple outcomes that are equilibriums: (B,B) and (O,O). However, both players would rather do something together than go to separate events, so no single individual has an incentive to deviate if others are conforming to an outcome: the man would attend the opera if he thinks the woman will be there even though he prefers the other equilibrium outcome in which both attend the baseball game. In this context, a consensus decision-making process can be considered as an instrument to choose the best strategy in a coordination game. The final decision is often not the first preference of each individual in the group and they may not even like the final result. But it is a decision to which they all consent because it is the best for the group. Consequently, a common final decision is achievable only if the man and the woman have explicit information on the decision of other member, then only if there is cooperation. If the man and the woman do not decide together where spend their time in the evening, probably, the result of implicit information and consequent false consensus effect will be that the man will go to the opera because he thinks that she decides to go there, and the woman will go to the baseball match to meet the man.

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