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TAX EVASION: A GAME COUNTERMEASURE

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ABSTRACT. We propose a game-theoretic model analyzing the interaction between the State and any possible relative taxpayer, by using a realistic probability (frequency) approach to the checking evasion strategy. Starting from Allingham and Sandmo's model (1972), we study a possible measure to prevent tax evasion and we also propose a "honesty-award" for Taxpayers declaring their entire income by using two Kalai-Smorodinsky solutions. This methodology leaves room for further development of the model, leading to a self-identification by tax evaders and honest citizens.

1. Introduction

Tax evasion is a phenomenon that has reached a monumental size for several reasons (Gober and Burns 1997; Richardson 2006). According to Murphy (2012), the European average of tax lost as a proportion of tax income is about 22% (see Fig. 1), and the total European shadow economy is more than 2000 billions of euros (Fig. 2). In this paper we propose - starting from the same methodologies developed by Allingham and Sandmo (1972) for their model - a general game-theory model that analyzes the interaction between the government and any possible relative tax-payer (for the methodologies used we refer the reader to Arthanari, Carfì, and Musolino 2015; Carfì and Musolino 2011a,b, 2012b, 2013b,c, 2014a,b; Musolino 2012).

2. Literature review

In this paper we shall refer to a wide variety of literature. First of all, we shall consider some papers on the complete study of differentiable games and related mathematical backgrounds, introduced and applied to economic theories since 2006 by Carfì and coworkers (see Agreste, Carfì, and Ricciardello 2012; Baglieri, Carfì, and Dagnino 2010, 2012; Carfì 2006b, 2008d, 2009a,b,c,f,g, 2010a,b, 2011b, 2012; Carfì and Fici 2012; Carfì, Gambarelli, and Uristani 2013; Carfì and Lanzafame 2013; Carfì, Magaudda, and

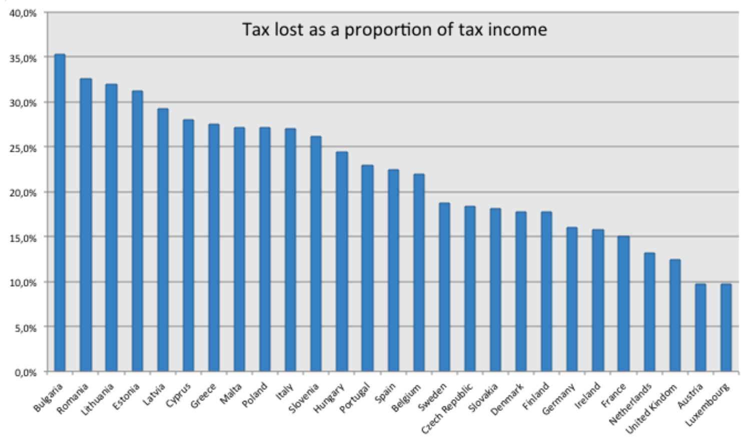

FIGURE 1. Tax lost as a proportion of tax income for the EU States.

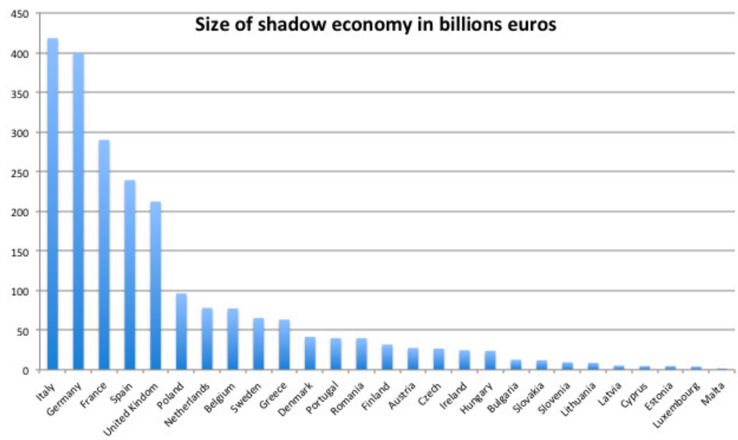

FIGURE 2. Size in billions Euros of shadow economy for the EU States.

Schilirò 2010; Carfì and Perrone 2012a,b, 2013; Carfì and Pintaudi 2012; Carfì and Ricciardello 2009, 2010, 2012a,b, 2013a,b; Carfì and Schilirò 2012a,b,c,d, 2013).

Specific applications of the previous methodologies, also strictly related to the present model, have been illustrated by Carfì and Musolino (2011a,b, 2012a,b,c, 2013a,b,c, 2014a,b, 2015). Other important applications of the complete examination methodology were introduced by Carfì and coauthors (Agreste, Carfì, and Ricciardello 2012; Arthanari, Carfì, and Musolino 2015; Baglieri, Carfì, and Dagnino 2012, 2015; Carfì 2012; Carfì and Fici 2012; Carfì, Gambarelli, and Uristani 2013; Carfì and Lanzafame 2013; Carfì, Patanè, and Pellegrino 2011; Carfì and Perrone 2011a,b,c, 2012a,b, 2013; Carfì and Pintaudi 2012; Carfì and Ricciardello 2012a,b, 2013a,b; Carfì and Romeo 2015; Carfì and Schilirò 2011a,b,c, 2012a,b,c,d, 2013, 2014a,b; Carfì and Trunfio 2011; Okura and Carfì 2014).

General ideas on the possible future applications of the methodologies introduced in the previous works could be deviced under the view of the researches carried out by Carfì (2004a,b,c,d, 2006a,c, 2007a,b, 2008a,b,c,e,f, 2009d,e, 2011a), Carfì and Caristi (2008), and Carfì and Cvetko-Vah (2011). Many of these researches and results were presented and discussed at the "Permanent International Session of Research Seminars" which was held at the University of Messina (see Carfì, Musolino, Ricciardello, and Schilirò 2012; Carfì, Musolino, Schilirò, and Strati 2013). In addition, in this paper we shall make systematic use of the researches carried out in the field of economy by Allingham and Sandmo (1972), Gober and Burns (1997), Murphy (2012), Musolino (2012), and Richardson (2006).

3. Description of the game

First player is the State, which chooses a percentage $x \in [0,1]$ of tax declarations to investigate and, consequently, also the probability to track down the tax evaders.

Second player is a Taxpayer that chooses a percentage $y \in [0,1]$ of his income R to declare.

Assumption 1 (potential tax without evasion). The State obtains, in absence of tax evasion, an income aR, where

- R is the total income of the Taxpayer;
- a is the tax rate cashed by the State on the Taxpayer's income.

Assumption 2 (actual tax). The State has

- the probability $1-x$ to obtain the income yaR, where y is the percentage of income R declared.
- the probability x to obtain the income yaR addicted to difference between
  - the penalty P paid by the Taxpayer on the not-declared income,
  - the total cost C for the State of its strategy to investigate the tax declarations.

Assumption 3 (tax penalty). The penalty P paid by the Taxpayer is given by

$$P(y) = na(1-y)R, \quad (1)$$

where

- $na > a$ is a coefficient representing the not-declared income $(1-y)R$ paid as penalty by the Taxpayer;
- $(1-y)$ is the percentage of not-declared income R.

Assumption 4 (cost for the State). The total cost C for the State is given by

$$C(x) = cx, \tag{2}$$

where

- c is the cost for the investigation of all tax declarations;
- x is the percentage of tax declarations investigated.

4. Payoff functions of the game

4.1. Payoff function of the State. It is given by summing

- the difference between the income that it effectively obtains and the income that it would obtain in absence of tax evasion (see assumptions 1 and 2);
- the difference between the income about the tax penalty and the cost of its strategy to investigate the tax declarations (see assumptions 3 and 4).

To obtain the function f_1, we use the von Neumann method, only with respect to the first strategy space {0,1} of the State. We consider - for every strategy y - the mixed extension of the finite stochastic variable $L(y) : \{0,1\} \to \mathbb{R}$, defined by

$$L(y)(0) = f_1(0,y) = ayR - aR$$

and

$$L(y)(1) = f_1(1,y) = ayR - aR + na(1-y)R - c,$$

by using the probabilistic scenarios only for the actions of the State. So, we have

$$\begin{aligned} f_1(x,y) &= \mathbb{E}_{(1-x,x)}(L(y)) = \\ &= \mathbb{E}_{(1-x,x)}(ayR - aR, ayR + na(1-y)R - cx - aR), \end{aligned}$$

and we obtain

$$f_1(x,y) = (aR(nx - nxy + y) - cx^2) - aR. \tag{3}$$

4.2. Payoff function of the Taxpayer. It is given by the difference between

- the tax that it would pay declaring all his income minus the tax that he actually pays (assumptions 1 and 2);
- the penalty paid by the Taxpayer about the not-declared income (assumption 3).

By adopting the von Neumann method, as before, we obtain:

$$\begin{aligned} f_2(x,y) &= \mathbb{E}_{(1-x,x)}(aR(1-y), aR - (ayR + na(1-y)R)) = \\ &= (aR(-nx + nxy - y)) + aR, \end{aligned} \tag{4}$$

for every (x,y).

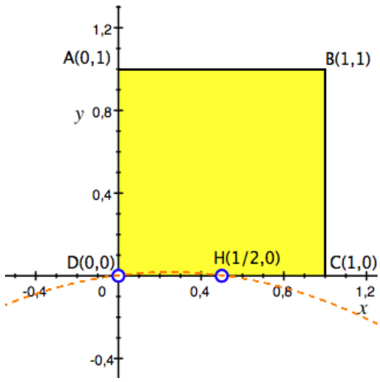

FIGURE 3. The critical area of the game

4.3. Payoff function of the game. It is given by

$$\begin{aligned} f(x,y) &= g(x,y) + aR(-1,1) = \\ &= (aR(nx - nxy + y) - cx^2, aR(-nx + nxy - y)) + aR(-1,1). \end{aligned}$$

We study only the kernel g, since any information on the game $(f,>)$ comes from the game $(g,>)$, by translation.

5. Payoff space

Since we are dealing with a non-linear game, it is necessary to study in the bi-win space also the points of the critical zone that belong to the bi-strategy space.

Critical space of the game. In order to find the critical area of the game, we consider the Jacobian matrix and we put its determinant equal 0.

About the gradients of f_1 and f_2, we have:

$$\text{grad } g_1 = (naR(1-y) - 2cx, aR(-nx+1))$$

$$\text{grad } g_2 = (naR(1-y), aR(-nx-1)).$$

After the calculations, the critical space of the game is:

$$Z_f = \{(x,y) : y = 2aRcx(1-nx)\}.$$

Assuming that $a = 1/4$, $n = 2$, $c = 1/4$ and $R = 1$, we obtain

$$Z_f = \{(x,y) : y = (1/8)x(1-2x)\}.$$

The critical area of our bi-strategy space is represented in Fig. 3 by the segment $[D,H]$.

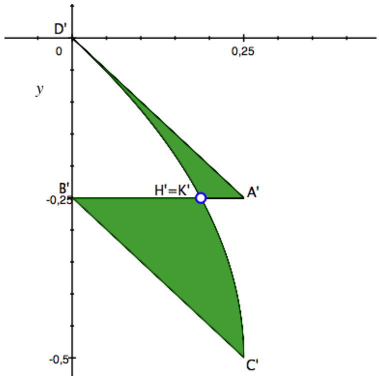

FIGURE 4. The payoff space

Payoff space. Transforming by g the sides of the square $E \times F$ and the critical space of the game $(g, >)$, we get the payoff space $g(E \times F)$ (Fig. 4).

Remark. For sake of illustration, we have chosen $a = 1/4$, $n = 2$, $c = 1/4$ and $R = 1$.

6. Nash equilibria

The best reply of State is

$$B_1(y) = \begin{cases} \{1\} & \text{if} \quad y \leq 1 - 2c/(naR) \\ \{naR(1-y)/2c)\} & \text{if} \quad y > 1 - 2c/(naR) \end{cases} .$$

The best reply of Taxpayer is

$$B_2(x) = \begin{cases} \{1\} & \text{if} \quad x > 1/n \\ \{0\} & \text{if} \quad x < 1/n \\ E & \text{if} \quad x = 1/n \end{cases} . \tag{5}$$

In Fig. 5 we have in red the inverse graph of B_1, and in blue that one of B_2 (we assume $a = 1/4$, $n = 2$, $c = 1/4$ and $R = 1$).

The Nash equilibrium is:

$$\mathrm{Eq}(B_1, B_2) = \left(\frac{1}{n}, 1 - \frac{2c}{an^2 R} \right) .$$

Analysis of Nash equilibrium. The Nash equilibrium is not on the proper maximal Pareto boundary. Moreover, it is not ethically a good equilibrium, because the Taxpayer tries to get smart declaring only a part of his income.

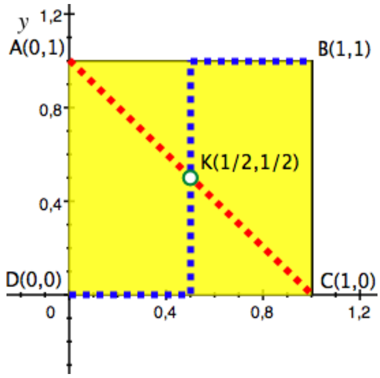

FIGURE 5. Nash equilibrium of the game

7. Defensive phase

Conservative value of a player: it is defined as the maximization of its worst win function.

Conservative value of the State: it is $v_1^\sharp = \sup_E g_1^\sharp$, where $g_1^\sharp$ is the worst win function of the State, and it is given by $g_1^\sharp(x) = \inf_{y \in F} g_1(x,y)$. Since the worst offensive strategies of the Taxpayer are

$$O_2(x) = \begin{cases} \{0\} & \text{if} \quad x < 1/n \\ \{1\} & \text{if} \quad x > 1/n \\ E & \text{if} \quad x = 1/n \end{cases},$$

we obtain:

$$g_1^\sharp(x) = \begin{cases} naRx - cx^2 & \text{if} \quad x \leq 1/n \\ -cx^2 + aR & \text{if} \quad x \geq 1/n \end{cases}.$$

So the conservative strategy of State is given by $x_\sharp = 1/n$, and its conservative value is

$$\begin{aligned} v_1^\sharp &= \sup_{x \in E} \inf_{y \in F} aR(nx - nxy + y) - cx^2 = \\ &= aR - (c/n^2). \end{aligned} \tag{6}$$

Conservative value of the Taxpayer. It is given by $v_2^\sharp = \sup_F g_2^\sharp$.

Since the offensive strategies of the State are

$$O_1(y) = \begin{cases} \{1\} & \text{if} \quad y < 1 \\ E & \text{if} \quad y = 1 \end{cases},$$

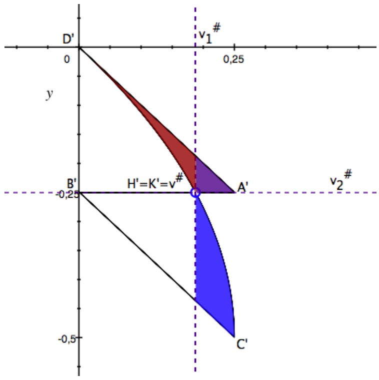

FIGURE 6. The conservative part of the game in the payoff space.

we obtain

$$g_2^\sharp(y) = \begin{cases} aR(-n+ny-y) & \text{if} \quad y<1 \\ -aR & \text{if} \quad y=1 \end{cases}.$$

Hence, the conservative strategy of Taxpayer is given by $y_\sharp = 1$, and its conservative value is:

$$\begin{aligned} v_2^\sharp &= \sup_{y\in F}\inf_{x\in E} aR(-nx+nxy-y) = \\ &= -aR. \end{aligned} \tag{7}$$

Therefore, choosing $R=1$, $a=0.25$, $c=1/4$ and $n=2$, the conservative bi-value is

$$v_g^\sharp = (v_1^\sharp, v_2^\sharp) = (3/16, -1/4).$$

In Fig. 6 we can see in red the conservative part of the Government in the payoff space, and in blue the conservative part of the Taxpayer. In purple is represented the shared conservative part.

Conservative cross: it is represented by the bi-strategies $(x_\sharp, y_\sharp)$, that is $H=(1/n,1)$. The conservative cross is not on the maximal Pareto boundary, but it represents a good compromise between the State and the Taxpayer. In fact, because of fear of being tracked down, the Taxpayer declares all his income ($y=1$). At the same time, the State plays the conservative strategy $x=1/n$ to guarantee itself an income of at least $W_1 = aR-(c/n^2)$.

Core of the game: the core is the part of the maximal Pareto boundary contained in the upper cone of $v_g^\sharp$. Therefore, we have

$$core'(G) = [L', A'],$$

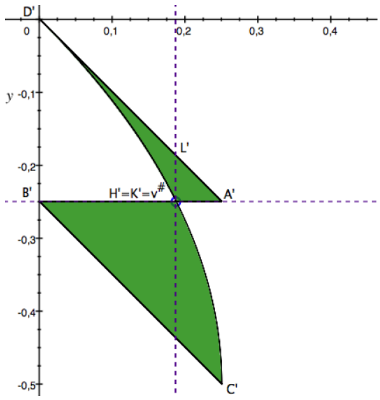

FIGURE 7. The $core'(G)$ of the game G

whose reciprocal image is

$$core(G) = [L,A],$$

that is represented in Fig. 7.

8. Possible normative countermeasures

Recalling Eq. 5, the State (in order to induce the Taxpayer to declare all his income) has the possibility to prevent the tax evasion in two ways:

- by its strategy x;
- by modifying the penalty coefficient n.

Assuming the State has structural reasons that prevent an effective movement of the public machine to modifying the strategy x, in a short time (for example Italy), the unique solution is to intervene on the penalty coefficient n. If this penalty coefficient is 4 instead of 2 (in the previous numerical case we have chosen $n = 2$), we obtain Fig. 8. We note that both Nash equilibrium and conservative value (in the game Nash equilibrium is equal to $v^\sharp$) are closer to the proper maximal Pareto boundary than the precedent numerical case. So, playing Nash or conservative strategies, the State obtain a greater income, while the Taxpayer has not any loss than before.

Moreover, recalling that

$$B_2(x) = \begin{cases} \{1\} & \text{if} \quad x > 1/n \\ \{0\} & \text{if} \quad x < 1/n \\ E & \text{if} \quad x = 1/n \end{cases},$$

it's enough $x > 1/4$ (lower than the previous $x > 1/2$) for the State in order to force the Taxpayer to declare all his income.

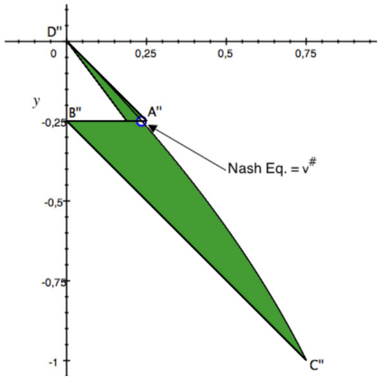

FIGURE 8. The payoff space with $n = 4$

9. Cooperative solutions: a possible "honesty-award"

Another way to avoid the presence of tax evaders is a cooperative agreement between the two players.

"**Honesty award**" **proposal**. We propose that the two player arrive to the point $A = (0, 1)$ - on the proper maximal Pareto boundary - with the promise by the State to give a "honesty-award" to the Taxpayer if he declares all his income, without investigating the declarations (in fact the State has to play $x = 0$ to arrive in A). This "honesty-award" corresponds to the division of the maximum collective profit according to Kalai-Smorodisky solutions.

Transferable utility solutions. We propose two Kalai-Smorodisky solutions, maximizing the collective payoff, with the payoff $K' = (aR - c/n^2, -aR)$ of the Nash equilibrium as threat point, and the supremum of the game or of the core as utopia points (Fig. 9), (see also Carfì and Musolino 2012a, 2013a, for this methodology). To find the points P' and P'' we have to resolve the equation systems between

- the straight line representing the proper maximal Pareto boundary, that is $y = -x$;
- the straight line joining the threat point K' and the utopia point $(aR, 0)$, that is: $y = \frac{aRn^2}{c}(-aR + x)$;
- the straight line joining the threat point K' and the utopia point $(aR, -aR + c/n^2)$, that is: $y = x - 2aR + c/n^2$.

Hence, as a possible "honesty award" we obtain:

$$P' = \left(\frac{(aRn)^2}{aRn^2 + c}, -\frac{(aRn)^2}{aRn^2 + c} \right), \tag{8}$$

and

$$P'' = \left(aR - \frac{c}{2n^2}, -aR + \frac{c}{2n^2} \right). \tag{9}$$

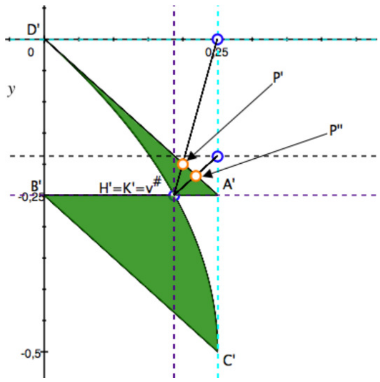

FIGURE 9. Cooperative solutions

10. Conclusions

In this paper we model, in a general and applicable framework, the interaction between the State and any possible relative Taxpayer, by using a realistic probability (frequency) approach to the checking evasion strategy. We show that:

(1) according to the conservative behavior, the Taxpayer has no convenience to declare an income inferior than the real one;
(2) according to the Nash equilibrium, the Taxpayer does not declare all his income, but the bi-profits are the same of the conservative behavior;
(3) the State can prevent the presence of tax evaders with a sufficiently high percentage (that is inversely proportional to the penalty coefficient) of tax declarations investigated; if the State increases the penalty coefficient, it obtains:
   - an approach of the Nash equilibrium and of conservative value to the maximal Pareto boundary with the highest collective profit,
   - a decrease in the minimum percentage of tax declaration to investigate in order to prevent tax evasion;
(4) two compromise solutions can be applied by the State in order to convince the Taxpayer not to evade taxes in exchange for "honesty-award";
(5) the possibility to require the honesty-award by taxpayers can help the State to find the tax evaders (who do not require the honesty-award for fear that the State investigates his declaration).

This methodology leaves room for further development of the model, leading to a self-identification by the tax evaders and honest citizens.

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