

TOBIN'S "A DYNAMIC AGGREGATIVE MODEL": AN ANALYTICAL INTERPRETATION

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ABSTRACT. In the attempt of arguing against the conclusion achieved by Harrod and Domar on the stability of growth, Tobin proposes in 1955 a "Dynamic Aggregative Model" which takes into consideration a monetary sector and a neoclassical two factors production function. Despite this model has received along the years a more accurate and advanced elaboration thanks to the theoretical developments achieved by Tobin on the analysis of the portfolio choices, on the investment decision and on the methodology which formalises the interactions between real and financial markets, the 1955 framework remains a benchmark for the specification of the financial sector in much of the Tobin's work in macroeconomic theory. However, in the light of the more advanced works developed by Tobin, the 1955's model presents some specification problems that have not been noticed in the literature.

1. Introduction

The contributions of Domar (1946) and Harrod (1939) on economic growth triggered a good deal of research at that time. Two were the principal lines of inquiry in this field. The first one developed a dynamic analysis of the disequilibrium. A second line of research focused on a dynamic analysis of equilibrium firstly proposed in 1955 Tobin's essay "A Dynamic Aggregative Model", which has become a seminal paper (Tobin 1955, p. 103). After, this latter line of inquiry on the dynamics of equilibrium at macro level was developed by other authors along the neoclassical tradition as Solow, Swan and Meade. Tobin's contribution marks, in fact, the beginning of a line of thought in the literature on the role of the financial activities in the growth models, tracing a furrow so deep that Johnson (1967) and Solow (1956) and later Phelps (1986), have recognised it as a cornerstone in this field:

"We hope that this article could celebrate the aggregative dynamic model elaborated by James Tobin a long time ago and to try to break the obsessive spell that this article has woven on me" (Phelps 1986, p. 3)

Almost all Tobin's work on the economic consequences of a fiscal and monetary policy in the short and in the long-run in the 70's and in the 80's,¹ is characterised by a systematic and integrated presence of some constitutive elements envisaged in his 1955's seminal paper. This model has received along the years a more accurate and advanced elaboration thanks to the theoretical developments achieved by Tobin on the analysis of the portfolio choice, of

¹About Tobin's macro-financial models (see Tobin 1975, 1978, 1982a,b, 1984, 1987).

the investment decision and on the methodology to formalise the interactions between real and financial markets. In the 1955's paper Tobin anticipates an informal description of the portfolio choices, but he does not make any explicit assumption on the investment function and only sketches the equilibrium conditions both in the real and in the financial markets that foresee the formulation of his 1968's article with Brainard. The basic model outlined in this article of 1955, however, remains a benchmark for the specification of the financial sector in much of the Tobin's work in macroeconomic theory.

Because of the pionic nature of the 1955's contribution and in the light of the more advanced work developed by Tobin, however, the model discussed in "A Dynamic Aggregative Model" presents some specification problems that have not been noticed in the literature. The main aim of the present note therefore is to rewrite the model proposed in 1955 bearing in mind the analysis developed by Tobin in the 70's and in the 80's and in particular in his *Nobel Lecture* (1981) (see Tobin 1982b). On the basis of this reconstruction we will argue that the model introduced by this economist in the 1955 is incomplete.

2. A Dynamic Aggregative Model

One of the main objective of Tobin in his 1955's contribution was to argue against the conclusion achieved by Harrod and Domar on the stability of growth, as he explained later (Tobin 1971):

"The second purpose of the article was to argue that capitalist economies are to good deal dark robust with respect to capital deepening than the growth models popular at the time suggested. According to the models of Harrod, Domar and Hicks, expansion at full employment was impossible if the saving-investment potential of the economy was I know great that the stock of capital would grow faster that the effective supply of labour. Capital would become technologically redundant; its installments of return would collapse; investment would decline, taking income and employment with it. The soughts to point out that this bleak picture could be considerably modified if to production function allowing variable proportions and to monetary sector were introduced" (Tobin 1971, p. 113)

Tobin's framework relies on four "*building blocks*" *i*) the saving function; *ii*) the production function; *iii*) asset preferences (liquidity preference) and *iv*) the labour supply-conditions. Combining these constitutive elements with informal considerations, Tobin examines the dynamic equilibrium properties of the model and develops some exercises of comparative dynamics and of disequilibrium in order to reconsider the problem of stability set by Harrod and Domar.

In the present work, with the purpose to make Tobin's model of 1955 consistent with the formulation of the macroeconomic framework presented in his *Nobel Lecture* "Money and finance in the macroeconomic process" (Tobin 1982b), the earliaer Tobin's analytical framework will be here reformulated reorganising the four constitutive elements of the model in three blocks representing the equilibrium conditions in the good, in the financial and in the labour markets respectively. In this perspective it is possible to say that the Tobin's earlier work considers the existence in the economy of three sectors (household,

firm and government): the first holder of equities and money. Equities, representative of quotas of physical capital, are issued by the firms and constitute the only way to finance real investment. Firms, moreover, distribute all profits to the holders of equities. The third sector is burdened by a debt equal to the value of the stock of money in circulation.

The equilibrium condition in the good markets represents for Tobin the first "building block" of the model. This was described simply by the equation $I = sY$, where I represents the net investment measured in real terms, s the propensity to save of the household and Y the real income. Such formulation does not take into account the influence of wealth and other variables on savings and does not make explicit an investment function. The latter is determined as the difference between income and consumption, under the hypothesis that any output not consumed is an addition to capital stock. More importantly, the equilibrium conditions do not explicitly consider the public sector, despite in the analysis developed by Tobin it is assumed to exist. In the 1955 model, in fact, a deficit may exist due to the transfers to the household sector, Tr . These transfers are determined exogenously and influence the available income of the households sector. The existence of public deficit therefore implies a simultaneous flow of new money issued by the public sector.

Taking into account the previous considerations the equilibrium condition in the good market may be reformulated in the following way:

$$I + Tr = s[Y + Tr] \quad (1)$$

$$I = \frac{dK}{dt} \quad (2)$$

$$Tr = Tr^* \quad (3)$$

where K indicates the quantity of capital in the economy.

The analysis concerning the equilibrium conditions in the financial markets represents, without any doubt, the most innovative part of the paper. In the light of the most recent elaboration, however, it appears, as we will try of to show in what follows, incomplete. The stock of money in circulation, according to Tobin:

"is exogenously determined and can be varied only by budget deficits and surpluses" (Tobin 1955, p. 105)

Such stock must grow in a dynamic equilibrium, to the same rate of growth of the whole economy, g , taking into account the variation of the general level of the prices. Considering that public deficits are due only to transfers to the household and indicating with H the stock of money in nominal terms and with P the price of the only existing good in terms of money, we can specify the equations related to the money supply in the following way:

$$P Tr = (g + \pi)H \quad (4)$$

$$\pi = \frac{dP/dt}{P} \quad (5)$$

where π indicates the rate of inflation. Such equations were not considered by Tobin amongst the *building blocks* of the model. Tobin, however, will make reference to these conditions later in the paper when he analyses different regimes of balanced growth that the economy can follow. The discussion of these different cases, which correspond to different hypotheses on the way the public deficit may be financed, will introduce in the analysis some inaccuracies, apparently neglected in the literature.

The supply of capital is derived from a production function with decreasing returns, assuming a inverse monotonic relation between the stock of capital and his marginal product. Indicating with r the return of the capital, we can write this equation as:

$$K = K(r) \quad \text{where} \quad K'(r) < 0. \quad (6)$$

The equations related to the demand of money and capital can be expressed, as suggested by Tobin, as a portfolio choices. When Tobin wrote his work in 1955 he had not yet published his famous article "Liquidity preference as behaviour towards risk", published later in 1958 in the *Review of Economic Studies*, that constitutes a cornerstone in modern theory of the portfolio analysis. Nevertheless, in this context, Tobin gives an informal description of such theory perfectly in line with the theory he will discuss in 1958. The formulation of the demand of the two financial activities, however, do not appear as clear as the intuition developed in the paper. First of all, Tobin introduces only one demand function, and in particular that one related to the money demand, including such equation amongst the "building blocks" of the model (Tobin 1955, pp. 95-98). Such equation assumes the form

$$M = PL(r, K, Y).$$

Second, Tobin introduces amongst the arguments of the function the rate of return of the capital but not the real return of money, and the amount of existing physical capital in the economy instead of wealth (Tobin 1955, p. 95).

Following the formulation adopted by Tobin in the Nobel Lecture (1981) (see Tobin 1982b), we rewrite this part of the model as:

$$H/P = L_h(r_h, r_k, Y, W) \quad (7)$$

$$K = L_k(r_h, r_k, Y, W) \quad (8)$$

$$W = H/P + K \quad (9)$$

$$r_h = -\pi \quad (10)$$

where Eq.(7) and Eq.(8) represent the demand for money and for quotas of real capital respectively both measured in real terms; W is financial wealth measured in real terms which is equal to the sum of the capital and the stock of money and r_h indicates the real rate of return of money, equal to the inverse of the rate of inflation.

As far as the equilibrium conditions in the labour market is concerned, Tobin specifies only the demand function derived from the equality condition between labour marginal product and the quantity of job employed. The supply of labour instead was not introduced. It may depend on the real wage or it can be inelastic. In a condition of a balanced growth, however, it must grow at the same rate of the whole economy, independent from the value assumed by the real wage. Such rate, in absence of technical progress is equal to the rate of natural growth of the available work force. We can write the conditions of equilibrium of the labour market in the following way:

$$N = N(w/p) \quad (11)$$

$$N = N_0 e^{gt} \quad (12)$$

$$g = n \quad (13)$$

where N indicates the demand and the supply of labour in the economy, N_0 the amount of labour available at the initial time of the process, w the money wage, and n the natural rate

of growth of the economy. In this new formulation the model contains thirteen unknowns $[I, Tr, Y, K, g, P, H, r_h, r_k, \pi, W, N, w]$ and the thirteen equations [1 – 13] are sufficient in number to fully determine the equilibrium value of the endogenous variables. Tobin discusses firstly in his paper an incomplete version of the model under the assumption of stationarity (Tobin 1955, p. 107) that is to say $n = 0$. Then he examines two cases of balanced growth:²

- a) the absence of inflation [$r_h = 0$];
- b) the existence of deflation with a rate equals to $-n$ or [$r_h = \pi = -n$] and the absence of variations in the nominal stock of money. In such way, the rate of growth of the stock of money in real terms is still equal to n , since:

$$\frac{dH/dt}{H} - \frac{dP/dt}{P} = 0 - (-n) = n$$

However, the introduction of the hypotheses relative to the last two cases generate problems of indeterminacy because they introduce in the model some constraints on the value of r_h and r_k without introducing at the same time others unknown variables in the system. On the base of our knowledge it is possible say that the existence of these problems has not been noticed in the literature so far, since such a literature has focused primarily on the real aspects of the model.

Surely it was not noticed in 1955 by Tobin himself, that didn't introduce in his article the model that has been presented here in a complete form.

3. Concluding remarks

Tobin's "A Dynamic Aggregative Model" (see Tobin 1955) constitutes a cornerstone in the literature on the role of the financial activities in the growth models. It also represents a benchmark for almost Tobin's work on the economic consequences of a fiscal and monetary policy in the short and in the long-run in the 70's and in the 80's.

In the light of the more advanced work developed by this economist, however, it is possible to say that the model discussed in the 1955's paper presents some specification problems. The existence of these problems has not been noticed in the literature that has focused primarily on the real aspects of the model. The present paper has tried to fill the gap, providing a new version of the model in a more complete form.

Surely Tobin did not notice in 1955 these specification problems, so he did not introduce in his seminal work the analytical solutions suggested in this paper.

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²Balance growth means proportional growth of capital, income and employment and implies, according with production function assumed in the model, constance of capital rent r_k and the real wage w . Tobin (1965, 1968) proposed similar monetary growth models.

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