DOI: 10.1478/AAPP.92S1B3

AAPP | Atti della Accademia Peloritana dei Pericolanti
*Classe di Scienze Fisiche, Matematiche e Naturali*
ISSN 1825-1242

Vol. 92, Suppl. No. 1, B3 (2014)

# ART & FINANCE: FINE ART DERIVATIVES

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ABSTRACT. This work is intended to introduce a new kind of asset, the so called *art asset*. This *financial* tool is an asset whose value is related to an art-work, and in particular to the artist reputation. It will be shown the evaluation of an art asset by using a particular kind of volatility, the $\alpha$-hedging. This tool normalizes the prices volatility of the art-works of an artist (or an art-movement) by a *sentiment index* referred to the Art Market. At last we shall show how the art assets' values are related to an *art*-call option.

## 1. Introduction

The Art Market is thought of as a solely aesthetic-dividend market par excellence. Robertson and Chong (2008) say that «the phrase 'aesthetic dividend' was used to describe the pleasure derived from owning art, but all too often it sounded like an uncomfortable apology for the fact that art, unlike 'real' investments, produced no income». But, as noticed by Robertson and Chong (2008) and Robertson (2005), the Art Market has becoming a more and more attractive motive to invest because of its monetary-returns besides its aesthetic value.

Nowadays the Art Market has drawn the attention of several investors, rather from 2012 the investments in art has raised of 69%. Moreover, Deloitte and ArtTactic (2013) show that the global investment in Art Founds increased in the year 2011/2012 from \$960 million to \$1.62 billion, and this is keep growing. Moreover, it is not a chance that the Art Market rapidly growing has been nourished just during the 2007-2013 crisis. This is because of the features of the Art Market, those characteristics which lure and warrant big investments. Yet, it is useful to outline the most important features:

- in a research carried out in 2003 by the Glenmede Trust Company (Robertson and Chong 2008) «fine art has shown a durable record of *price retention* and a *low correlation* to more conventional asset classes», this means that an art asset could be profitably used for a well-diversified portfolio. Rather a higher initial volatility will be a lower volatility in the long-term, vice versa for financial markets.
- Art has weaker performance in periods of low inflation and weak growth ——the exact point in the business cycle when equity returns are at their strongest (Robertson and Chong 2008). Yet again it means a very good diversification across economic

scenarios. Thus, the Barclays Capital study recommended an allocation of up to 10 per cent of a diversified portfolio in fine art (Robertson and Chong 2008).

- Moreover, there are significant tax breaks. Thus, under some provisos hinging on Countries' laws, there is an artists's exemption from income tax.
- Art Markets provide an exceptional risk-return ratio.
- The buy-side is heavily concentrated in high net worth individuals (HNWIs) and collectors. Art dealers, corporations and museums are also major buyers in the market (Sommer 2012). This means that the investments are uncorrelated to the mid-income fluctuations, and thus investments can be done even in crisis scenarios.

But we have to warn that all that glitters is not gold. There are clearly hurdles which prevent some inverstors from investing in art, moslty during 1980-1990. Let us summarize the features of Art Markets, but this time we shall focus on their bad taste:

- It is needed a good investemt plan programming because there is a problem of *liquidity*. This is because the art investments are usually undertook on Art Funds, that are similar to investment funds but they are made of *objets d'art*, paintings et cetera. An Art Fund collects these works of art and once the investment period is over, the *Lock-in*, it will start the sell-process. This last process is a very thorny problem, because if the Art Fund has unsold works of art in its stock, even when the sell-process is through, it has to close out its work for lower prices facing up a significant loss.
- There is a problem of conflict of interest. Art dealers, Art Advisors and other Art Market players can affect the pice of art's price, and this can be done by their personal advantage because, for example, an art dealer invests in art too and her has the opportunity to get richer tainting the price level.
- Transitional costs may be very high, from 10% to 20% of the hammer price.
- Low trasparency and low rules characterized Art Markets.

The bad-taste features have been the incentive to try to put into effect a more transparent, efficient and reliable market of art's works. This endeavour has been undertaken by Deloitte Luxembourg and ArtTactic, which aim to build up the *Art&Finance industry*. It will be a very interesting investment center provided by high transparency given the *on-line* investment approach and thanks to the Grand Duke of Luxembourg, which, by *the Luxembourg freeport*, will establish a physical place wherein the art's works can be collected freed from any surtax and transitional costs.

The investments in Art have two motives: on the one hand they are still led by the aesthetic attraction, but on the other hand their long-term return and the above-mentioned good-features lure income motives.

Beyond the pramount effort of Deloitte Luxembourg and ArtTactic, it is worth noting that two world leading auction houses such as Sotheby's and Christie's, have developed high level educational programs in order to increase the competence of Art students and practitioners towards a professional Art Business knowledge.

The environment depicted, albeit roughly, in this section, shows that the Art Market as income-motive investment is growing year by year. The attention that world leading institutions have directet to Art Markets during these years is a valuable proof of the ongoing growth of this new business sector.

This paper is devoted to the theoretical environment wherein an art asset price floats. Without generality it is important to understand that the present work intends the price of an art-work strictly hinged on *drift* and *volatility* with respect to the artist and not to the piece of art itself. We want to stress the mathematical framework of this pricing by using a particular kind of volatility, the $\alpha$-hedging (see Section **2**). In Section **3**, it will be shown the fundamental equation known as Black-Scholes-Merton, but it will be $\alpha$-volatility-adjusted.

## 2. The $\alpha$-hedging

When one is talking about Art Markets in comparison with Financial Markets, the first thing to understand is that the *art assets* do not follow the traditional analysis of volatility returns. Rather, it is of utmost importance to grasp the confidence of art investors in the art market. This is one of the most important variables which determines if a piece of art will be worth either a high value or a low one. For a work of art, the mainstream appreciation is not grabbed by a mere volatility estimation, thus it is needed a tool which shows how the time series return volatility is affected by nowadays mainstream art investors. We call this kind of volatility $\alpha$-hedging, in short ${}^{h}\alpha$.

Because of the Lehman Brothers collapse in September 2008, the art market prices fell to 23.5% in 2009 (Mei Moses index), and market liquidity evaporated as auction sales fell by up to 80% from their peak (Deloitte and ArtTactic 2013). Furthermore «the ArtTactic Art Market Confidence Index dropped 40% in November 2007, ten months prior to the actual art market collapse. This signalled at the time that almost half of the survey respondents were negative on the six-month art market outlook. This shift in sentiment among 'experts' in the contemporary art market triggered ArtTactic's interest in trying to better understand the relationship between the art market sentiment and the buying and selling patterns in the market place.» (Deloitte and ArtTactic 2013). The ArtTactic Art Market Confidence Index is an index which measures the market sentiments about an Art Market branch; Deloitte and ArtTactic (2013) say that «The initial sample for the U.S. and European contemporary art market was carefully constructed over a period of three years, finally reaching just over 100 individuals, ranging from private collectors to art advisors, curators, galleries, museums, and also more investment-oriented buyers. By capturing the consensus opinion of these experts, it gave us a better tool to understand their level (or lack) of support for a particular artist and his/her market. The sample was weighted by giving different players more or fewer votes depending on their position in the art market hierarchy». Moreover «Every six months, ArtTactic sends a questionnaire on contemporary art market conditions to 100 key art market experts. Responses to several of the survey questions related to the economy and the primary and secondary art markets are combined to form the ArtTactic Art Market Confidence Index (AAMCI). In addition to the overall confidence index, ArtTactic also produces confidence indexes on 250 individual U.S. and European, Indian and Chinese modern and contemporary artists.» (Deloitte and ArtTactic 2013).

The $\alpha$-hedging tells us that the return volatility $\sigma$ (as a standard deviation, for now it is better off not bothering to define *what kind* of volatility is) has to be considered with respect to the AAMCI (denoted by $\lambda_{\mathrm{A}}$) which, for clarity, is the confidence in an art-movement or in an artist himself and not in a single piece of art, so

$$ {}^{h}\alpha = \frac{\delta}{\lambda_{\mathrm{A}}} \times 100, \tag{1} $$

where $\lambda_{\mathrm{A}} = [0, 100]$. The meaning of Eq. (1) is straightforward: given a constant volatility (say $\sigma$-volatility), the AAMCI (or $\lambda_{\mathrm{A}}$) worsen the overall volatility (say $\alpha$-volatility) if $\lambda_{\mathrm{A}} \leq 99$, whereas leaves the $\sigma$-volatility unchanged if $\lambda_{\mathrm{A}} = 100$. Moreover, if $\lambda_{\mathrm{A}} \searrow 0$ then ${}^{h}\alpha \to \infty$, thus as the confidence in Art Markets approaches to zero, the $\alpha$-volatility becomes unmanageable, the risk of loss is too high, hence the investment must not be done. In particular, if $\lambda_{\mathrm{A}}$ approaches to zero, the loss is certain, even if the $\sigma$-volatility is not too high. The $\alpha$-volatility is an interesting characteristic of Art Markets.
It is possible to define $\sigma$ as *historical volatility*. Define $S_i$ as an art asset price in the interval $i = 0, 1, \dots, n$ with no dividend payments, and define $u_i$ as

$$ u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \quad \text{for} \quad i = 1, 2, \dots, n. \tag{2} $$

Given that $n+1$ is the number of observations, define $s$ (the $u$'s standard deviation) as

$$ s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} u_i\right)}. \tag{3} $$

By defining $\tau$ as the interval year-lenght

$$ \hat{\sigma} = \frac{s}{\sqrt{\tau}}. \tag{4} $$

This is the well known historical volatility definition, thus it is possible to define the $\alpha$-historical volatility as

$$ {}^{h}\hat{\alpha}_y = \frac{\hat{\sigma}}{\lambda_{\mathrm{Ay}}} \tag{5} $$

where, given a $y = 1, 2, \dots, m$, the $\lambda_{\mathrm{Ay}}$ is defined as the annual AAMCI referred to the year $y$ which is the year where $\tau$ is referred to. Thus if $\tau_y$ then $\lambda_{\mathrm{Ay}}$. Furthermore, given that $s$ is an estimation of $\delta\sqrt{\tau_y}$ (the standard deviation of the assets prices logarithm ratio $u_i$), there exists a standard error $\frac{\hat{\sigma}}{\sqrt{2n}}$.

Now we shall see the equation of the art assets price, and for the sake of simplicity it is possible, for now, to get rid of the $\alpha$-historical volatiliy which is an estimation of ${}^{h}\alpha$ and thus this simplification does not affect the following reasoning at all.

**2.1. Art assets log-normal prices.** In this section it is assumed that the instantaneous rate of variation in art asset prices follows a normal distribution. Moreover $\mathbb{E}$ is the expected return, whereas $\sigma_y$ the annual volatility which we shall denote by $\sigma$ if there is no peril of misunderstanding. We define a time $t$ and an interval $\Delta t$ wherein prices change with mean $\mathbb{E}\Delta t$ and standard deviation $\sigma\sqrt{t}$ (see, e.g., Hull 2011); hence

$$ \frac{\Delta S}{S} \sim \psi(\mathbb{E}\Delta t, \sigma^2 \Delta t) \tag{6} $$

where $\Delta S$ is the price variation in $\Delta t$. Now, define $S_T$ as the asset price at time $T > 0$, and define $S_0$ as the asset price at time 0. It is known that it implies

$$\ln(S_T) \sim \psi\left[\ln(S_0) + \left(\mathbb{E} - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right] \tag{7}$$

where $\sigma^2$ is the annual variance of pieces of art returns and $\mathbb{E}$ the expected value. It is important in understanding that $\sigma^2$ and $\mathbb{E}$ are defined with respect to the mean value of the pieces of art created by an artist, as a whole. This means that the price of a particular art-work is strictly related to the artist information, of course it is needed to cut the extremal values in order to normalize the estimation. Furthermore, as it is plain, it is not considered the *sentiments* determinant in this step, it is treated like a general asset. So, when does AAMCI come running? Given the normality condition, we say, for example, that there is a probability of 95% that the price of the art asset $S$ will float in the following interval

$$\left[\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)T\right] - 1,96 \times {}^h\alpha < \ln(S_T) < \left[\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)T\right] + 1,96 \times {}^h\alpha.$$

**Example.** We face a current art-work price of \$1.000 of an artist called $A$. The sell-buy process is ment to be executed in a secondary art market and it will be sell to an Art Fund. The expected annual return $\mathbb{E}$ is 20% per year. This return is intended to be the buy-sell processes during the last year before the present buy-sell, it is valued for all of the pieces of art sold by $A$ by cutting the worst and the best price. This 20% at $t$ is an expected return as a mean of the whole sell-buy processes executed in $t-1$ by $A$. The annual volatility is 30%, yet again it is a volatility measured for the whole sell-buy processes executed in $t-1$ by $A$, thus not with respect to the single work of art. Now, the $S_T$ price by 6 months is

$$\ln(S_T) \sim \psi\left[\ln(\$1.000) + \left(0,2 - \frac{0,3^2}{2}\right) \times 0,5, \quad 0,3^2 \times 0,5\right]$$

hence

$$\ln(S_T) \sim \psi\left[6.98, \quad 0.045\right].$$

Now $\sigma = \sqrt{0,045}$ then $\sigma = 0,21$, so is here that the AAMCI comes running. Assuming that the AAMCI=79%, thus

$$\begin{aligned} {}^h\hat{\alpha}_{0.5} &= \frac{\sigma}{\lambda_{A0.5}} \\ &= \frac{0,21}{0,79} \\ &= 0,2659 \end{aligned}$$

Now, assume a confidence interval of 95% and the normality condition, then

$$6,98 - 1,96 \times 0,2659 < \ln(S_T) < 6,98 + 1,96 \times 0,2659$$

or better

$$\$e^{6,98-1,96\times 0,2659} < S_T < \$e^{6,98+1,96\times 0,2659}$$

hence the pricing interval will be

$$\$638,32 < S_T < \$1.810,15$$

Had we not used the $\alpha$-hedging, by using $\sigma = 0,21$ we would have obtained a different interval such as

$$\$712,23 < S_T < \$1.622,30$$

Hence, there is a difference of $\$261,13$ between the given intervals' lengths and AAMCI = 79%.

## 3. The fundamental equation

In this section it will be shown the Black-Scholes-Merton equation for a call option price in terms of art assets. The assumptions followed in defining this equation are the following:

- The instantaneous log returns of the stock price is a geometric Brownian motion, where $\mathbb{E}$ and $\sigma$ are constants;
- short stales are allowed and it is possible to use their proceeds;
- an equal riskless rate $r$ for the whole expiration dates;
- the stock does not pay a dividend;
- no arbitrage opportunity;
- transactions do not incur any fees or costs;
- continuous trading.

Define $N(x)$ as

$$N(x) = \frac{1}{2\pi} \int_{-\infty}^{x} e^{\frac{y^2}{2}} dy$$

that is to say $N(x) = P[X \leq x]$ where $X \sim N(0,1)$. A *fair price* for an art call option is

$$C = S_0 N\left( \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{{}^h\alpha_T} \right) - Ke^{-rT} N\left( \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{{}^h\alpha_T} \right) \tag{8}$$

in a simpler fashion

$$C = S_0 N(d_{\alpha 1}) - Ke^{-rT} N(d_{\alpha 2}) \tag{9}$$

where $S_0$ is the asset price at time 0, $K$ is the strike price, $T$ is the maturity date, $r$ is the risk-free interest rate, $\sigma$ is the volatility of the whole art-works returns of an artist, say $A$, and ${}^h\alpha_T$ is the $\alpha$-hedging in $[t,T]$ with $t < T$. In Eq. (9), $N(d_{\alpha 1})$ is the present value of receiving the stock if the option will be in the money, whereas $N(d_{\alpha 2})$ is the present value of paying the exercise price in that event. For the sake of clarity the relationship between the $\alpha$-hadging and the volatility in Eq. 8 is denoted by the following equality

$$\frac{\sigma^2}{2} = ({}^h\alpha_T \times \text{AAMCI})^2.$$

## 4. Conclusions

This work has been written as an introductory and theoretical paper on the subject. It is a brand new topic which is very attractive to practitioners and it deserves more and more attention. Rather Eq. (8) can be used by insurance companies in order to evaluate an *index-linked* insurace and Eq. (7) can be thought of as a benchmark for a *unit-linked* insurace. In the latter case it will be needed, for the insurace, to be linked to an Art Fund index. Art assets may be a very interesting *safe-haven asset* uncorrelated to the financial world, hence it is needed to diversification portfolios in managing risks.

This paper has followed some general assumptions: first, a work-art's price is estimated with respect to the whole artist's features, thus it is assumed that we do not know (or that we are provided by few information about) the price time series of a particular piece of art. Rather an art-work can be sold in the primary art market (thus the first sell) or, for example, is traded in the secondary art market but it has been sold only two times. This is why I have used *drift* and *volatility* with respect to an artist and not to a particular piece of art. Of course if we are provided by a rich time series of the art-work prices, *drift* and *volatility* can be referred to the piece of art itself. Secondly, the operation, or at least a *quasi*-operation of the *Art&Finance industry* underpinned by the *Luxembourg free-port*, hence I assumed transparency and no transaction costs. This second assumption is important because of its extent: we need the operation of the Art&Finance industry so as to enhance Art Markets and their financial tools.

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Paper presented at the *Permanent International Session of Research Seminars* held at the DESMaS Department "Vilfredo Pareto" (Università degli Studi di Messina) under the patronage of the *Accademia Peloritana dei Pericolanti*.

Communicated 25 June 2012; published online 22 April 2014