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AN ANALYTICAL MODEL TO CALCULATE ABSORBED FRACTIONS FOR INTERNAL DOSIMETRY WITH ALPHA, BETA AND GAMMA EMITTERS

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ABSTRACT. We developed a general model for the calculation of absorbed fractions in ellipsoidal volumes of soft tissue uniformly filled with alpha, beta and gamma emitting radionuclides. The approach exploited Monte Carlo simulations with the Geant4 code to determine absorbed fractions in ellipsoids characterized by a wide range of dimensions and ellipticities, for monoenergetic emissions of each radiation type. The so-obtained absorbed fractions were put in an analytical relationship with the 'generalized radius', calculated as $3V/S$, where V is the ellipsoid volume and S its surface. Radiation-specific parametric functions were obtained in order to calculate the absorbed fraction of a given radiation in a generic ellipsoidal volume. The dose from a generic radionuclide can be calculated through a process of summation and integration over the whole radionuclide emission spectrum, profitably implemented in an electronic spreadsheet. We compared the results of our analytical calculation approach with those obtained from the OLINDA/EXM computer software, finding a good agreement in a wide range of sphere radii, for the high-energy pure beta emitter ^{90}Y, the commonly employed beta-gamma emitter ^{131}I, and the pure alpha emitter ^{213}Po. The generality of our approach makes it useful and easy to implement in clinical dosimetry calculations as well as in radiation safety estimations when doses from internal radionuclide uptake are to be taken into account.

1. Introduction

Internal radiotherapy makes wide use of radiopharmaceuticals labelled with beta-emitting radioisotopes for the treatment of tumours and various diseases. Radionuclides characterized by different average energies can be chosen to optimize the delivered dose distribution within target volumes, taking advantage of the different intensities of the cross-fire effect (i.e. the multiple irradiation of a target cell from radiations originating from different neighbouring points of accumulation of the radiopharmaceutical).

The knowledge of radiation absorbed fractions in organs and tissues is a fundamental item both in radionuclide internal dosimetry, a branch of medical physics concerning the assessment of radiation absorbed doses during nuclear medicine therapies with radionuclides and radiopharmaceuticals, and in radiological protection, particularly in the estimations of radiation doses from radionuclide intake in humans or non-human biota.

The absorbed fraction is defined as the fraction of energy emitted by the radionuclide source which is absorbed in the target volume. Following the approach introduced by the Medical Internal Radiation Dose (MIRD) Committee, the mean radiation absorbed dose to the target volume from a generic radiation type *i* can be evaluated as:

$$\overline{D}_i = \frac{\widetilde{A}\Delta_i \phi}{m} \tag{1}$$

where $\widetilde{A}$ is the cumulated activity (Bq s), i.e. the total number of disintegrations occurring in the source, m is the mass of the target volume, Δ_i is the average energy of the radiation emitted per disintegration (J Bq^{-1}) and ϕ is the absorbed fraction (Bolch *et al.* 2009).

The sphere model is often employed to represent small up-taking regions in human dosimetry, as well as whole non-human organisms for environmental radioactivity contamination estimations. Generalization to ellipsoids is desirable since the assumption of a spherical shape for an ellipsoidal structure leads to inaccuracies in dosimetric evaluations of the order of several per cent (Grošev *et al.* 2008), depending on the dimension, degree of non-sphericity, and radionuclide emission spectrum. The ellipsoidal shape is widely employed to model not only small target tissues such as thyroid nodules (Amato, Lizio, Ruggeri, *et al.* 2011), but also the thyroid remnant tissue after thyroidectomy (Grošev *et al.* 2008) and whole organs, such as kidneys (Bouchet *et al.* 2003). Ellipsoidal models are also extensively employed for environmental radiation safety purposes (Ulanovsky and Pröhl 2006).

Concerning electrons, supposed to be uniformly emitted within the target volume, when the target size is greater than the particle range, the absorbed fraction tends to unity. When, instead, this condition is not fulfilled, the absorbed fraction may be significantly lower and its dependence on target dimension and electrons energy must be considered.

Concerning X and gamma photons, since in the energy range of radionuclide gamma emission the cross sections of the dominant interaction mechanisms, i.e. photoelectric effect and Compton scattering, are strongly dependent upon photon energy, the absorbed fraction varies dramatically with energy.

Regarding alpha particles, the absorbed fraction shows a steeper slope between a few microns and hundreds microns of sphere radii, depending on the particle energy, as shown in the present work, while the absorbed fraction approaches to unity for sphere radii higher than 1 mm and energies higher than 5 MeV.

Aim of the present work was to expose our analytical approach for the calculation of absorbed fractions in a comprehensive form, and, through the development of a dedicated electronic spreadsheet, to compare the results of our method for spheres with the results of the OLINDA/EXM software.

2. Materials and methods

In the notation adopted by the MIRD Committee (Bolch *et al.* 2009), the mean radiation absorbed dose from a single monoenergetic emission of a radiation *j* to the target volume is:

$$D_j = \frac{\widetilde{A} n E \phi}{m} \tag{2}$$

where $\widetilde{A}$ is the time-integrated activity, i.e. the total number of disintegrations occurring in the source, which in our case coincides with the target volume, m is the target mass, E is the particle energy, n is the emission probability per disintegration, and ϕ is the absorbed fraction.

In the most general case of a radionuclide emitting alpha particles, monoenergetic and beta electrons, and photons, the overall dose to the target volume will be given by the sum of all radiation doses:

$$\overline{D} = D_\alpha + D_\beta + D_e + D_\gamma = \frac{\widetilde{A}}{m} E_{dep}, \tag{3}$$

$$\begin{aligned} E_{dep} = \sum n_{\alpha,i} E_{\alpha,i} \phi_{\alpha,i} + \int \frac{dn(E)}{dE} E \phi(E) dE + \\ \sum_i n_{e,i} E_{e,i} \phi_{e,i} + \sum_i n_{\gamma,i} E_{\gamma,i} \phi_{\gamma,i} \end{aligned} \tag{4}$$

where E_{dep} is the average energy deposition per disintegration, while n_α, n_e and n_γ are the monoenergetic alpha, electron (Auger or conversion) and photon (X or γ) emission probabilities, respectively, of energies E_α, E_e and E_γ. The quantities ϕ_α, ϕ_e and ϕ_γ are the alpha, electron and photon absorbed fractions, obtained from Eq. (4). The integral represents the contribution due to the continuous beta spectrum. This generalized model accounts for all the possible decay modes of the radionuclide considered.

Our analytical method allows to calculate the absorbed fractions for photons, electrons and alpha particles in ellipsoidal volumes (Amato, Lizio, and Baldari 2009a,b, 2011; Amato, Italiano, and Baldari 2013) and, consequently, to evaluate the doses from radionuclides uniformly distributed in such volumes. This method was developed from evaluation of absorbed fractions in multiple shapes and volumes of ellipsoids, for monoenergetic radiations of each type. They were calculated through Monte Carlo simulations carried out with the Geant4 (Agostinelli *et al.* 2003) code, which was extensively validated and applied in the field of medical dosimetry (Amato, Salamone, *et al.* 2013; Botta *et al.* 2013).

Soft tissue with 1.04 g cm^{-3} and atomic composition taken from (Valentin and Streffer 2002) was assumed for all the calculations.

In our model, for a generic ellipsoidal shape, characterized by a, b, c semiaxes, the ‘generalized radius’ is:

$$\rho = 3\frac{V}{S} \tag{5}$$

where V denotes the target volume and S its surface, respectively calculated as:

$$V = \frac{4}{3}\pi abc \tag{6}$$

and

$$S = 4\pi \left[\frac{(ab)^d + (bc)^d + (ac)^d}{3} \right]^{1/d} \tag{7}$$

where $d = \ln 3/\ln 2 \cong 8/5$. The approximated formula for S gives a relative error below 1.2% (Klamkin 1971, 1976). It is apparent that, for a spherical volume, ρ coincides with the sphere radius.

ELECTRONS [Eqs. (9-10)]			PHOTONS [Eqs. (11-12)]		ALPHA PARTICLES [Eq. (13)]	
d	2.7010	$\times 10^{-6}$	g	9.355×10^{-5}	f	1.008
f	1.6181		h	3.173×10^{-5}	g	3.1797
g	2.3790	$\times 10^{3}$	r	337.25	h	5.4741×10^{-7}
h	7.4563		m	3.336		
k	4.6835	$\times 10^{2}$	n	2.947		
m	1.9465	$\times 10^{3}$	a	1.118		
n	4.1747		b	10.036		
p	−4.6656	$\times 10^{2}$	c	1.269×10^{5}		
r	1.000133		d	−24.823		
			e	257.722		
			f	1.157×10^{5}		

TABLE 1. Values of the fit parameters introduced for the three particle types (Amato, Lizio, and Baldari 2009a,b; Amato, Lizio, Ruggeri, *et al.* 2011; Amato, Italiano, and Baldari 2013).

Amato, Lizio, and Baldari (2009a,b, 2011) and Amato, Italiano, and Baldari (2013) demonstrated that the absorbed fraction ϕ can be put in relationship with the generalized radius ρ(cm):

$$\phi(\rho) = \left(1 - \frac{\rho_0}{\rho^s}\right)^{-1} \tag{8}$$

where ρ_0 and s are two parameters depending on the particle type and energy.

The parameters $\rho = \rho_0(E)$ and $s = s(E)$ were fitted with suitable parametric functions, characteristic of the particle type. For electrons (Amato, Lizio, and Baldari 2011), the functions adopted were:

$$\rho_0(E) = dE^f \exp\left(-gE^{-\frac{\ln E}{\ln^2 10}} 10^{-\frac{h}{\ln E}}\right) \tag{9}$$

and

$$s(E) = \frac{kE - m}{E^r + n} + p. \tag{10}$$

For photons (Amato, Lizio, and Baldari 2009a), ρ_0 and s were fitted with the functions:

$$\rho_0(E) = \frac{gE^m}{1 - hE^n\left(1 - \exp - \frac{E}{r}\right)} \tag{11}$$

and

$$s(E) = \frac{aE^3 + bE^2 + E + c}{E^3 + dE^2 + eE + f} \tag{12}$$

In the case of alpha particles (Amato, Italiano, and Baldari 2013), the analytical form adopted for ρ_0 was:

$$\rho_0(E) = fg^{(\log E)^h} \tag{13}$$

while for s it was possible to assign a constant value, s = 1.393.

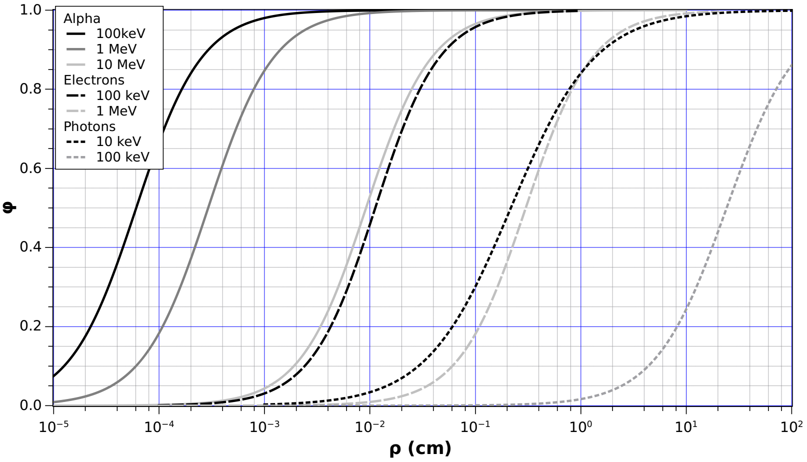

FIGURE 1. Alpha particle, electron and photon absorbed fractions as a function of the generalized radius: plot of the fit functions for some relevant energies.

A list of parameter values contained in Eqs. (9-13) is reported in Table 1. In order to be consistent with the original notation, we use for these parameters the original nomenclature, despite the same symbol can represent parameters of different radiations.

We implemented the above described formalism in a comprehensive electronic spreadsheet, in order to calculate the doses in a generic ellipsoidal shape, provided the radionuclide emission spectrum, the ellipsoid's semiaxes, and the cumulated activity in the volume.

We compared the results of the OLINDA/EXM software (Stabin, Sparks, and Crowe 2005), with the outcomes of our analytical method. OLINDA/EXM, an acronym for Organ Level INternal Dose Assessment/EXponential Modeling, is a software which calculates radiation doses to different organs of the body from systemically administered radiopharmaceuticals and performs regression analysis on user-supplied biokinetic data, to support such calculations. It includes a 'Sphere Model' in which radiation absorbed doses to spheres uniformly uptaking a radionuclide can be calculated, once the cumulated activity is obtained from the fit and integration of the time-activity curve, by means of the EXM module. Since OLINDA/EXM provides no means to calculate doses in ellipsoidal shapes, the comparison with our model was possible for spheres only, by assuming a cumulated activity of 1 MBqh.

3. Results and discussion

For the comparison, we considered a high-energy pure beta emitter such as ^{90}Y ($< E >_\beta$ = 948.8 keV, $T_{1/2}$=2.67 days), a commonly employed beta-gamma emitter such as ^{131}I ($< E >_\beta$ = 181.5 keV, $T_{1/2}$ = 8.02 days) and a pure alpha emitter, ^{213}Po ($< E >_\alpha$ = 8.376 MeV, $T_{1/2}$= = 4.2 μs). All secondary radiations were considered in the calculations; radiation decay data were obtained from (Stabin and Da Luz 2002). In order to compare the results for ^{90}Y and ^{131}I in a range of dimensions in which the absorbed fractions raise from a few percent to almost unity, we considered spheres ranging between 0.1 grams (0.29

Mass (g)	R (cm)	^{90}Y	ε^{90}Y (%)	^{131}I	ε^{131}I (%)	^{213}Po	ε^{213}Po (%)
0.1	0.29	2.68×10^{3}	−4.9	9.88×10^{2}	−5.0	4.80×10^{4}	−0.4
0.5	0.49	7.35×10^{2}	−3.3	2.15×10^{2}	0.5	9.62×10^{3}	−0.3
1	0.62	3.97×10^{2}	−3.2	1.09×10^{2}	−1.8	4.81×10^{3}	−0.2
2	0.78	2.09×10^{2}	−4.1	5.54×10^{1}	−1.4	2.41×10^{3}	−0.4
6	1.13	7.48×10^{1}	−3.9	1.88×10^{1}	−2.1	8.03×10^{2}	−0.4
10	1.34	4.64×10^{1}	−3.1	1.14×10^{1}	−2.6	4.82×10^{2}	0.0
20	1.68	2.41×10^{1}	−1.7	5.90	−0.7		
60	2.43	8.39	0.0	2.04	−0.5		
100	2.88	5.09	0.2	1.25	−0.8		
500	4.92	1.05	1.0	2.80×10^{-1}	1.8		
1000	6.20	5.30×10^{-1}	0.8	1.50×10^{-1}	4.2		

TABLE 2. Absorbed dose per unit activity (mGy/MBq) in spheres: comparison between our results and those from the OLINDA/EXM software. The symbol ε represents the relative per cent deviation of our result with respect to the reference value.

cm radius) and 1000 grams (6.2 cm radius), while for the very short-ranged alpha particles emitted by ^{213}Po a maximum mass of 10 grams (1.34 cm radius) was chosen.

Some examples of the analytical curves representing the absorbed fractions as a function of the generalized radius are represented in Fig. 1. It is worth noting that the same analytical form can accurately describe the behaviour of the absorbed fractions for primary particles characterized by different interaction mechanisms. Namely, alpha particles lose energy in tissue-equivalent media essentially through inelastic collisions, while electrons lose energy also via bremsstrahlung, and photons interact through photoelectric and Compton effects. The different orders of magnitudes in the particle range (alphas and electrons) and mean free path (photons) is the cause for the different cut-off generalized radii of each particle type.

In Table 2 we show the results of the comparison between radiation absorbed doses calculated by means of our analytical approach, described in the previous Section, and the tabular values obtained from the OLINDA/EXM computer software. The symbol ε represents the relative per cent error between our result and the reference value taken from the OLINDA/EXM software.

The agreement between our results and the OLINDA/EXM data ($-5\%/+1\%$ for beta-gamma emitters and $-0.4\%/0.0\%$ for the alpha emitter ^{213}Po) demonstrates once again the validity of our analytical model. A general systematic trend of underestimation of the absorbed doses with respect to the OLINDA/EXM data can be put in relationship with the difference in the densities of the soft tissues considered: while our model rely on simulations carried out on a soft tissue with 1.04 g cm^{-3} density, OLINDA/EXM assumes unitary density for the soft tissue. An analogous result has been already reported in the literature (Lanconelli *et al.* 2012).

We observe that the discrepancies for alpha emitters are appreciably lower than those for beta-gamma emitters: this can be explained by considering that, in the range of target dimensions considered, the absorbed fractions for electrons and photons vary considerably, while alpha absorbed fractions approach to unity. Consequently, all the discrepancies ascribed to the analytical model for the calculation of the absorbed fraction are enhanced for beta and gamma emitters, while they become nearly negligible for alpha emitters.

4. Conclusions

We developed an analytical model which allows a comprehensive calculation of absorbed fractions from alpha, beta and gamma emitting radionuclides, together with their daughters, uniformly distributed in ellipsoidal volumes of soft tissue (Valentin and Streffer 2002) of any ellipticity and volume, in the whole range of practical interest for internal dosimetry in nuclear medicine therapeutic applications, as well as in radiological protection estimations of doses from an internal contamination.

By exploiting the analytical forms for the absorbed fraction as a function of the generalized radius, and the parametric functions for cut-off radius ρ_0 and exponent s, whose forms and parameter values are characteristic of the radiation type, it is possible to calculate the doses to ellipsoidal shapes from a general radionuclide by means of a simple electronic spreadsheet.

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