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EXPERIMENTAL STUDY OF SYNCHRONIZATION EFFECTS IN A SYSTEM CONSTITUTED BY WEAKLY COUPLED METRONOMES: A DIDACTIC EXPERIMENT

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ABSTRACT. The aim of this paper is to describe the phenomenon of oscillators' synchronization through a didactic experiment that involves a system constituted by a set of weakly coupled oscillating metronomes. A metronome is a device that allows to vary and select the oscillation parameters and to investigate synchronization processes. To carry out the experiment, metronomes were set over a common platform disposed on cylinders arranged transversally and that were free to roll on a fixed plane. In this paper we shall describe a teaching unit addressed to students of an academic course in Physics, Mathematics or Engineering.

1. Introduction

Synchronization is a remarkable phenomenon observed, for example, when coupled oscillators, having different initial conditions, spontaneously change their oscillation parameters and synchronize their phases (Strogatz 2000; A. Pikovsky, M. Rosenblum, and Kurths 2001). Generally speaking, synchronization phenomena occur in a wide range of contexts, from biological systems to mechanical and electronic oscillators, making it a fundamental concept in both theoretical and applied physics and its study has broad implications across various scientific and engineering disciplines (M. G. Rosenblum, A. S. Pikovsky, and Kurths 1996; Strogatz 2003). To report a few examples, in physics it helps in understanding phenomena such as the coherent radiation of laser beams and the behavior of superconducting Josephson junctions; in biology, synchronization explains the coordinated contraction of cardiac cells that produce a heartbeat and the emergent behavior seen in colonies of social insects like ants and bees (Glass and Mackey 1988; Winfree 2001; Boccaletti *et al.* 2002). The use of metronomes on a common moving platform provides a clear, visual representation of synchronization phenomena. Their use allows to mimic the coupling seen in more complex systems on a scale and in a way that is manageable and understandable (Pantaleone 2002; Boda *et al.* 2013; Goldsztein, Nadeau, and Strogatz 2021). In particular, metronomes, which are in essence mechanical oscillators, when placed on a platform that can move, interact through mechanical vibrations and air movements, synchronizing their phases. The proposed experiment is mainly intended to integrate a mathematical description of the synchronization phenomenon with a simple experiment. By physically observing the synchronization of metronomes, students are visually faced with the concepts of phase change and energy transfer among coupled oscillators (Acebrón, Bonilla, and Spigler 2000; Zhang, Yu, and Wang 2017). Therefore, this integrated approach is supposed to be effective in helping students understand this complex phenomenon through a direct observation and a manipulation of the involved experimental parameters. The significance of this approach is therefore amplified by the dual focus on theoretical comprehension and empirical application. It not only demonstrates synchronization in a controlled setting but also encourages students to consider the underlying physics governing the behavior (Hu *et al.* 2013). Implementing this experiment in an academic setting - particularly in courses related to Physics, Engineering, and Mathematics - provides an invaluable tool for educators to engage students actively. It promotes a deeper understanding of dynamical systems theory and nonlinear dynamics, which are crucial for advanced scientific education and research. Moreover, this experiment fosters critical thinking and analytical skills as students hypothesize, experiment, and derive conclusions from real-world data (Caccamo and Serpe 2023; Serpe 2023). In summary, the synchronization of metronomes on a moving plane not only illustrates a complex physical phenomenon but also enhances educational methodologies by connecting theoretical concepts to practical demonstrations. This enriches the learning environment, making complex concepts more accessible and stimulating intellectual curiosity and scientific inquiry among students.

2. Theoretical framework

The concept of synchronization in coupled oscillators traces its roots back to the 17^{th} century when Christian Huygens, a Dutch scientist, first observed the phenomenon. While working on the design of pendulum clocks, Huygens (1673) noticed that two pendulum clocks mounted on a common support would, over time, end up swinging in perfect opposition to each other a phenomenon he termed "sympathy." His initial observations laid the groundwork for understanding how coupled oscillators can influence each other's motion through slight mechanical interactions, even in the absence of direct contact. In the 1970s, Yoshiki Kuramoto introduced a mathematical model that significantly advanced the understanding of synchronization (Kuramoto 1975). The Kuramoto model describes a system of coupled oscillators, each with its intrinsic natural frequency. Despite these differences, the oscillators can self-organize into a synchronized state due to a coupling force. This model is described by the equation

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)\,,$$

where θ_i is the phase of the i^{th} oscillator, ω_i is its natural frequency, is the coupling strength, K is the total number of oscillators, and the summation is taken over all other oscillators in the system. This simple yet powerful model has been applied to different fields, from Biology to Engineering illustrating the universal nature of synchronization. The mathematical treatment of synchronization explores how individual differences in oscillator frequencies can be overcome by coupling, leading to a collective rhythm. The analysis often involves examining the stability of the synchronized state, determining how changes in parameters like the coupling strength K and the distribution of natural frequencies affect the system's behavior. Techniques from nonlinear dynamics and statistical physics are employed to predict critical thresholds for synchronization and to explore the dynamics near these critical points. Understanding these theoretical aspects is crucial for designing the experimental setup involving metronomes on a moving plane (Acebrón *et al.* 2005; Chavez, Hwang, and Boccaletti 2007). By varying parameters such as the number of metronomes, their intrinsic frequencies (set by adjusting the metronome weights and pendulum lengths), and the flexibility of the plane (affecting how motion is transmitted between the metronomes), different synchronization regimes can be explored (Winfree 1967; Abrams and Strogatz 2004; Rodrigues *et al.* 2016). The experimental setup acts as a real-world representation of the Kuramoto model, providing a tangible method for observing theoretical predictions. From the theoretical framework, we expect that as the coupling (interaction via the moving plane) between metronomes increases, the system will move from a state of desynchronization (where each metronome shows its own phase independently) to one of synchronization (where they show the same phase). This transition, dependent on the coupling strength and the diversity of natural frequencies, offers rich learning opportunities. Students can observe first-hand how varying parameters affect the system's behavior, gaining insights into the complex interplay between individual components' properties and the collective dynamics (Néda *et al.* 2000; Strogatz *et al.* 2005; Breakspear, Heitmann, and Daffertshofer 2010).

3. Experimental setup

The experimental setup is reported in Figure 1. It includes a platform, supported by two cylindrical rollers, which permits slight horizontal movements.

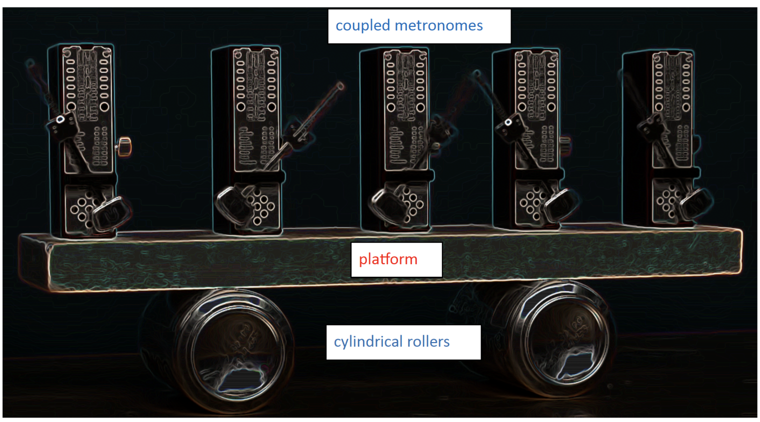

FIGURE 1. Platform supported by two cylindrical rollers which allows horizontal movements.

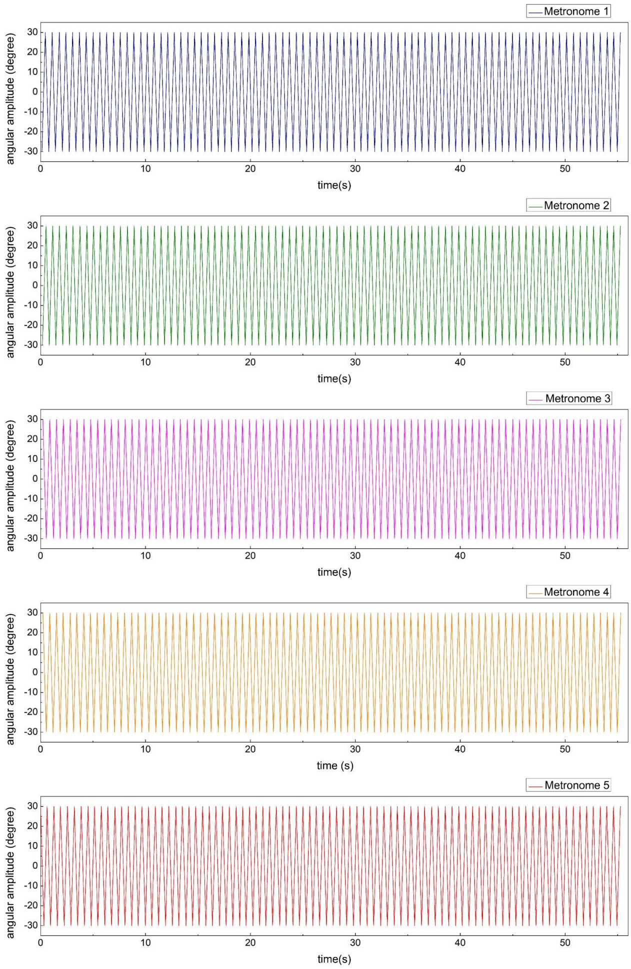

FIGURE 2. Synchronization of the five metronomes over a 55-second interval. The displacement as a function of time of metronome 1 is reported as a blue curve, of metronome 2 as a red curve, of metronome 3 as a pink curve, of metronome 4 as a yellow curve, of metronome 5 as a red curve, This time interval captures the transition towards the synchronization dynamical state.

The materials for the platform and the cylinders are lightweight metals which ensure smooth motion and minimal resistance, which is essential for observing the interactions between the metronomes. In the present experiment, five metronomes are used. The spacing between metronomes is adjusted to ensure that while the metronomes are far enough apart to initially operate independently, they are close enough for the motion of the platform to influence their synchronization over time. The exact distance can vary based on the experiment's goals. At each metronome oscillation, a force onto the platform results causing it to move slightly. This movement of the platform acts as a medium through which energy and momentum are transferred between the metronomes. Over time, this interaction leads to a mutual adjustment of phases until the metronomes synchronize. Observing this progression from independence to synchronization provides insights into the dynamics of coupled oscillators. To capture the synchronization process, the setup included measurement tools like high speed cameras. These instruments allow to record the initial phase, the frequency of oscillation, and the synchronization of the metronomes. Data collected from these tools are then analyzed to study the phase relationship. In particular, the experiment consisted of activating in sequence the five metronomes with different initial phases but with equal maximum oscillation angle. The starting time of the experiment was the time when all the metronomes oscillate. The final time of the experiment was the instant when all the metronomes oscillate with the same phase. Figure 2 reports the plots of the registered metronome amplitudes as a function of time in the time range when all the metronomes, with different initial phase, are activated.

In order to appreciate the different initial phases of the five metronomes, Figure 3 shows the initial registered behavior of the angular oscillation amplitude versus time, while Figure 4 shows the registered behavior in the final time range when all the metronomes are synchronized. As it can be seen, as time progresses, due to the coupling effect among the metronomes realized by the moving platform they are placed on, the oscillations phase differences decrease, and the metronomes swing in unison (Acebrón *et al.* 2005; Dörfler and Bullo 2014; A. Pikovsky and M. Rosenblum 2015).

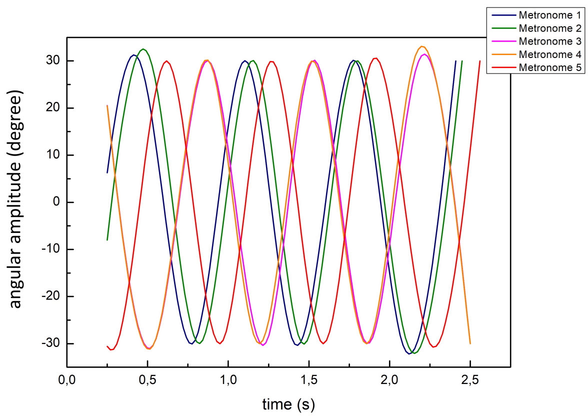

FIGURE 3. Initial registered behavior of angular oscillation amplitudes versus time.

4. Data analysis

After data collection, math software tools allows to analyze the phase relationships between the metronomes over time. Finally, statistical methods assess the repeatability of results across multiple trials and help determine the reliability of observed synchronization patterns.

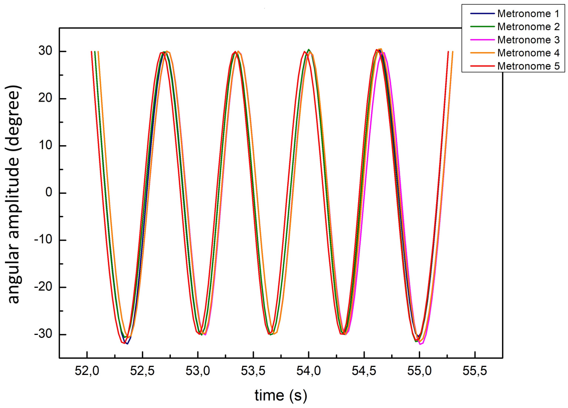

FIGURE 4. Registered behavior in the final time range when all the metronomes are synchronized.

Figure 5 shows, as an example, the behavior of phase difference between the first and the fifth metronome. As it can be seen the phase difference, starting from the non-zero initial value, follows an exponential trend, towards the zero. In particular, the decaying exponential curve reveals that after about 30 seconds the two metronomes reach the synchronized oscillation regime.

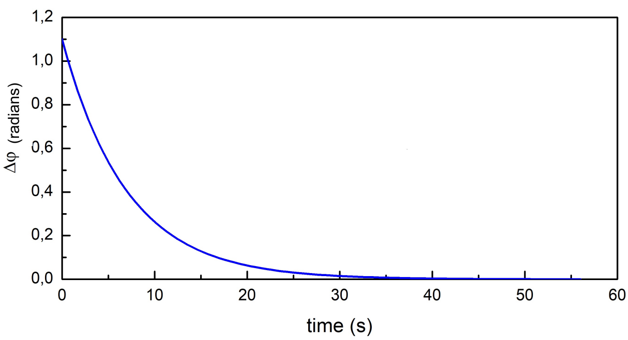

FIGURE 5. Behavior of the phase difference between the first and fifth metronome over time. The phase difference, expressed in radians, shows a decrease from an initial non-zero value towards the zero, indicating synchronization after approximately 30 seconds.

5. Conclusions

This study is addressed to clarify the synchronization phenomenon using a simple experimental setup constituted by metronomes moving on a common moving platform. The experiment visually and quantitatively confirms that taking into account different initial conditions, coupled oscillators, represented by metronomes, synchronize their phases over time due to interactions mediated by a shared moving platform. The results validate theoretical models of synchronization such as the Kuramoto model, showing real-world applicability in a controlled setting. The educational impact of integrating theoretical and experimental approaches in this study was significant: students who participated in, or observed, the experiment reported a better understanding of synchronization. Students learned to handle and interpret real-time data, connecting theoretical predictions with empirical observations. The interactive nature of the experiment, requiring active participation and observation, significantly increased student engagement. Hands-on experiences are known to boost curiosity and motivation, making learning more enjoyable and impactful. The findings of this experiment reaffirm the value of integrating experimental approaches in education, particularly in fields involving complex physical phenomena. In conclusion, the metronome synchronization experiment underscores the importance of experiential learning, where students can bridge the gap between theoretical knowledge and practical application. This approach is essential for educating future scientists and engineers, equipping them with the tools necessary to tackle complex challenges in their respective fields.

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