DOI: 10.1478/AAPP.1001A4

AAPP | Atti della Accademia Peloritana dei Pericolanti
Classe di Scienze Fisiche, Matematiche e Naturali
ISSN 1825-1242

Vol. 100, No. 1, A4 (2022)

AN INTERVAL-STOCHASTIC COMPROMISE PROGRAMMING FOR URBAN WATER RESOURCE ALLOCATION

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ABSTRACT. As water demand is exponentially increasing and supplies are scarce, integrated urban water management is an important task in all cities and regions. It is a multi-objective problem, because of the many objectives and criteria that should be analyzed. The key goal of this research was to create an integrated decision-making-based multistage scenario-based interval-stochastic programming (IDMSISP) model for urban water management. IDMSISP is a multistage optimization method with two objective functions that can reflect dynamics, uncertainty, and risk analysis in water resource management. This model was developed for the case of study, based on compromise programming. Two prominent objectives involving the value of the benefit of the network and social satisfaction level are examined. The capacity and environment constraints are considered under uncertainty. The uncertainty in these constraints was solved by applying the chance constraint method.

1. Introduction

With rapid population growth and increasing urbanization, the demand for urban water is increasing, but water supplies are limited and diminishing. Consequently, it is necessary to develop an efficient urban water management plan that emphasizes the integrated management of the practical features of water services. Many studies in the last decades have focused on water resource allocation and management (Li *et al.* 2009; Zhang *et al.* 2018; Khosrojerdi *et al.* 2019; Li *et al.* 2019; Martinsen *et al.* 2019; Zhang *et al.* 2019; Zhang and Xi 2020). Jacovkis *et al.* (1989) constructed a linear-programming multi-objective model for the analysis and planning of water resource systems. A mixed-integer programming model was introduced by Srinivasan *et al.* (1999) for the operation of a water supply reservoir and optimization of its proficiency. Srdjevic (2007) developed a decentralized decision problems model for water management. The model is based on the analytic hierarchy process (AHP) and its application in subgroups. Zarghami and Szidarovszky (2009) reported a decision model for water supply management. They designated how demand management mechanisms are fundamental in water saving. However, many system parameters in water resource management problems are uncertain. These uncertainties, which appear as constraints,

objectives, etc., should be addressed in many different formats in the water management system. Hence, many studies have focused on presenting uncertainties in different formats in water resource management systems (Zare M. and Daneshmand 1995; Seifi and Hipel 2001).

For uncertain input data that can be signified by chance or probabilities, chance-constrained programming (CCP) and multistage stochastic programming (MSP) methods have been developed for decision problems. CCP, as one of the robust approaches, can handle uncertainties expressed in system constraints in stochastic water resource management. Edirisinghe *et al.* (2000) presented a model based on CCP for the planning of reservoir capacity. Xu *et al.* (2009) suggested a stochastic robust chance-constraint programming (SRCCP) model for coping with different uncertainties in the environmental, economic, and system-reliability waste-management approach. Guo and Huang (2009) constructed a two-stage fuzzy chance-constrained programming method for handling uncertainties in water resource management systems.

Another effective approach to manage uncertainty, especially in decision-making processes, is modeling the problem through MSP. It is useful for dealing with uncertainties presented as a probability distribution and for allowing updated decisions at each time point based on the knowledge of sequentially realized uncertain situations (Dupačová 2002; Li *et al.* 2008; Birge and Louveaux 2011). Housh *et al.* (2013) introduced a limited multi-stage stochastic programming (LMSP) for a multi-stage water supply system operation problem to optimize flow and salinity decisions. The proposed model is based on a decision-clustering framework, which identifies the optimal decisions between similar decisions. To deal with uncertainties expressed as probability distributions, discrete intervals, and fuzzy sets, Khosrojerdi *et al.* (2019) proposed a two-stage interval-parameter stochastic fuzzy programming method. Li *et al.* (2008) constructed a multistage scenario-based interval-stochastic programming (MSISP) method for water-resource allocation under uncertainty. It has the qualities and abilities to consider the risk of violating the system constraints under multiple uncertainties. Uncertainties are displayed as probability distributions and interval values to be efficiently included within the optimization framework. It can demonstrate the dynamics of system uncertainties and decisions made under a representative set of scenarios. Almost all the previous studies that considered uncertainty in the different formats of their models used a single objective model. However, many objectives in water resource management must be considered in water-related urban procedures, for example, environmental issues, quality of the water, and human wellbeing. Optimizing a single objective normally occurs to the detriment of another objective.

An adaptive decision-making process for urban water management is discussed in this article. We develop a multi-stage scenario-based interval-stochastic programming and use a compromise programming model to solve the multi-objective problem. The model applies MSP and inexact chance-constrained programming (ICCP), which allow uncertainties and dynamics in the system to be considered through this framework. The remainder of this paper is organized as follows: The methodology is described in Section **2**, and the case study is outlined in Section **3**. In Section **4**, we formulate our methods. Section **5** provides the results and discussion. Notations and definitions are introduced in Section **6**. We conclude the paper with Section **7**, where we provide our outcomes.

2. Methodology

2.1. Compromise programming model. To evaluate multiple-objective problems, compromise programming (CP), based on the concept of distance, is applied. It is a method for identifying solutions that are the nearest to the optimal one based on a distance metric. Among the possible scores for the i criterion, there is one value that is favored over the others (Zeleny 1982). When working with objective functions $f(x)$ that are not represented in incommensurable units, a scaling function is used to ensure that all objective functions have the same range, which is called normalizing (Abrishamchi *et al.* 2005). The distance parameter is written as follows if the scaling function is linear:

$$L_p(x) = \left[\sum_{i=1}^{I} w_i^p \left[\frac{f_i^* - f(x)}{f_i^* - f_{i^*}} \right]^p \right]^{1/p}. \tag{1}$$

where the weight w_i signifies the importance of objective i relative to the other objectives. f_i^* and f_{i^*} are the best and worse values for criterion i, respectively. The decision makers' concern about the maximum deviation is reflected by the parameter p.

2.2. Interval mathematical programming and chance constraint programming. To deal with uncertainties in systems, interval mathematical programming (IMP) was developed. Where the available data are inadequate for creating a distribution function, each variable in an inexact set has upper and lower bounds that are used to estimate uncertainties. Only the right-hand side can be reliably reflected by IMP. Other right-hand side parameters that are random must also be mirrored. For coping with this form of uncertainty and evaluating the risk of breaching the uncertain constraints, the chance-constrained programming (CCP) approach can be used (Charnes *et al.* 1971; Charnes and Cooper 1983).

The CCP method converts a stochastic linear problem into a deterministic version by setting a certain level of probability q_i, signifying the acceptable risk of violating the uncertain constraint i. The constraint should be satisfied with a probability level of at least $1 - q_i$. As a result, the set of possible solutions is constrained by the following:

$$\begin{aligned} &Pr[\{A_i(t)X \le b_i(t)\}] \ge 1 - q_i, \\ &A_i(t) \in A(t), \quad i = 1, 2, \ldots, m, \end{aligned} \tag{2a}$$

where the decision variables are denoted by vector X, and $t \in T$ signifies the random elements in the $A(t)$, $B(t)$, and $C(t)$ sets (Huang 1998; Li *et al.* 2006). Constraint (2a), which is nonlinear, can be rendered linear by applying the following formula:

$$A_i(t)X \le b_i(t)^{q_i}, \quad \forall i, \tag{2b}$$

where $b_i(t)^{q_i} = F_i^{-1}(q_i)$ is the cumulative distribution function (CDF) of b_i. However, the interval parameter programming (IPP) technique should be implemented into the CCP system to address some limitations of the CCP method, such as uncertainties in left-hand

sides and cost/revenue parameters in the objective function (Infanger 1992; Zare M. and Daneshmand 1995; Huang 1998).

The interval-parameter chance-constrained programming (ICCP) model is:

$$\text{Max} \quad f^{\pm} = C^{\pm}X^{\pm}, \tag{3a}$$

subject to,

$$Pr[\{A_i(t)X^{\pm} \leq b_i(t)^{\pm}\}] \geq 1 - q_i, \tag{3b}$$

$$A_i(t)X^{\pm} \in A(t)^{\pm}, \quad i = 1,2,\ldots,m, \tag{3c}$$

$$x_j^{\pm} \geq 0, x^{\pm} \in X^{\pm}, j = 1,2,\ldots,n,. \tag{3d}$$

where $A^{\pm} \in \{R^{\pm}\}^{m\times n}$, $C^{\pm} \in \{R^{\pm}\}^{1\times n}$, $X^{\pm} \in \{R^{\pm}\}^{n\times 1}$, and $R^{\pm}$ denote a set of interval numbers (Li *et al.* 2008). A number with defined lower and upper limits but unknown distribution details defines an interval value (Huang 1998). Model (3) can then be transformed into a deterministic version using the following formula:

$$\text{Max} \quad f^{\pm} = C^{\pm}X^{\pm}, \tag{4a}$$

subject to,

$$A_i^{\pm}(t)X^{\pm} < B^{\pm}(t)^q, \quad A_i^{\pm}(t) \in A^{\pm}(t), \quad i = 1,2,\ldots,m, \tag{4b}$$

$$x_j^{\pm} \geq 0, x_j^{\pm} \in X^{\pm}, j = 1,2,\ldots,n. \tag{4c}$$

where $B^{\pm}(t)^q = \{b^{\pm}(t)^{q_i}, \quad i = 1,2,\ldots,m\}$. The combination of ICCP and MSP leads to a multi-stage scenario-based interval-stochastic programming (MSISP) model to cope with uncertainty in the constraints as well as interval values in the objective function (Li *et al.* 2008).

3. Case study

The major water resources are two dams, both of which are supplied by surface water. Ground water and wells are the other water resources. Dam surface water is treated for purification, whereas ground water does not need to be purified due to its natural purity. To purify surface water, two treatments are used. Then, both types of water are transported to the reservoir before being pumped to the consumption zone. Two random stream flows exist: the first flows into dams and the second flows into the reservoir. During the planning horizon, stream flows are random variables, and the relevant water allocation plan is dynamic (Fig. 1).

4. The proposed model

4.1. The objective functions. The maximization of the net benefit (NB) and maximizing the level of social satisfaction (SL) were considered the two main objective functions.

- Maximizing of the net benefit (NB): maximizing the predicted net system benefit from water resource allocation. If the water is delivered as promised, it will benefit

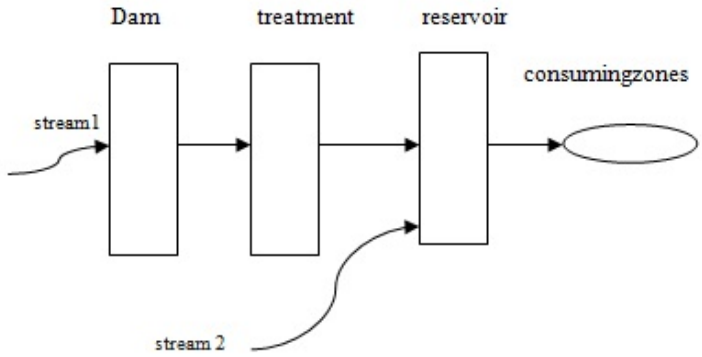

FIGURE 1. An illustration model of HPWN.

the local economy. If not, water must be supplied from other sources or demand must be reduced, with both cases resulting in penalties for the local economy (Li *et al.* 2008). This objective is stated as:

$$\mathrm{Max} f^{\pm} = \sum_{t=1}^{T} NB_t^{\pm} H_t - \sum_{t=1}^{T} \sum_{k_1=1}^{k_1^t} \sum_{k_2=1}^{k_2^t} p_{tk_1} p_{tk_2} PE_t^{\pm} Y_{tk_1k_2}^{\pm},$$

where $NB_t^{\pm}$ denotes the net gain per unit of water distributed over time period t and PE_t is penalty when water is not delivered. In period t, H_t is the water allocation goal promised to the consuming areas. $Y_{tk_1k_2}^{\pm}$ denotes the shortage level over period t under scenarios k_1 and k_2, which are correlated with joint probabilities of $p_{tk_1} p_{tk_2}$, which are linked to the random water availability of the streams.

- Maximizing the level of social satisfaction (SL): maximizing the level of social satisfaction for each water resource. There are three main criteria for assessing the degree of social satisfaction: the quality of drinking water, the amount of the water source, and the socio-economic impacts. The following is the objective function:

$$\mathrm{Max} SL^{\pm} = \sum_{t=1}^{T} \sum_{k_1=1}^{k_1^t} \sum_{d=1}^{2} \sum_{r=1}^{2} SW_r X_{tk_1d}^{\pm} + \sum_{t=1}^{T} \sum_{k_2=1}^{k_2^t} SWC_{tk_2}^{\pm},$$

where SW_r, $r = 1, 2$ and SW are the social satisfaction degree related to water resources and wells, respectively, computed by Fattahi and Fayyaz (2010) obtained by AHP; X denotes the amount of water transferred from water resources to treatments; and C is amount of water transferred from wells to the reservoir.

4.2. Model constraints.

4.2.1. *Water mass balance constraint.* Constraints (6a), (6b), and (6c) show the water resources' mass balance, water treatments, and water reservoir in each time period, respectively.

$$M_{r(t+1)k_1} = M_{rtk_1} + Q^{\pm}_{rtk_1} - \sum_{d=1}^{2} X^{\pm}_{rdtk_1} UX_{rd} - E^{\pm}_{r}, \tag{6a}$$

$$TT_{d(t+1)k_1} = TT_{dtk_1} + \sum_{d=1}^{2} X^{\pm}_{rdtk_1} UX_{rd} - \sum_{d=1}^{2} L^{\pm}_{dtk_1}, \tag{6b}$$

$$MM_{(t+1)k_1k_2} = MM_{tk_1k_2} + C^{\pm}_{tk_2} - \sum_{d=1}^{2} T^{\pm}_{dtk_1k_2} + \sum_{d=1}^{2} L^{\pm}_{dtk_1}, \tag{6c}$$

where under scenario k_1, Q_{rtk_1}, which reaches dams, is a random inflow into stream 1 in period t. The allowed paths are introduced using the UX parameter, which is a 0–1 value. The treatments collect and distribute potable water from the water supply to the reservoir, and the water allocation to the consuming zones is distributed from the reservoir.

4.2.2. *Available water constraint.*

$$H^{\pm}_{t} - Y^{\pm}_{tk_1k_2} \le T^{\pm}_{tk_1k_2}, \ \ \forall t; \ \ \forall k_1,\ldots,k^t_1; \ \ \forall k_2,\ldots,k^t_2, \tag{6d}$$

Constraint (6d) signifies that the amount of water allocated to the consumers must not exceed the amount of water released from the reservoir.

4.2.3. *Water allocation target (consuming zone) constraint.*

$$De_{t\min} \le H_t \le De_{t\max}, \quad \forall t, \tag{6e}$$

$$De_{t\min} \le T^{\pm}_{tk_1k_2} \le De_{t\max}, \quad \forall t, \tag{6f}$$

Constraints (6e) and (6f) indicate that allocated water must be sufficient to meet the user's minimum needs while not exceeding their maximum needs.

4.2.4. *Reservoir capacity constraint.*

$$Pr = \{TT_{dtk_1} \le TM(d)^{\pm}, \quad t; \ \ \forall k_1,\ldots,k^t_1; \ \ d = 1,2\} \ge 1-q, \tag{6g}$$

$$Pr = \{MM_{k_1k_2} \le MZ^{\pm}, t; \ \ k_1,\ldots,k^t_1; \ \ k_2,\ldots,k^t_2\} \ge 1-q. \tag{6h}$$

The storage volume must not surpass the reservoir capacity in all scenarios according to constraints (6g) and (6h). The two constraints, which are nonlinear in general, can be converted into the linear constraints as:

$$TT_{dtk_1} \le (TM(d)^{\pm})^q, \ \ t; \ \ k_1,\ldots,k^t_1; \ \ d = 1,2, \tag{6i}$$

$$MM_{k_1k_2} \le (MZ^{\pm})^q, \ \ t; \ \ k_1,\ldots,k^t_1; \ \ k_2,\ldots,k^t_2. \tag{6j}$$

These constraints are linear, and $F^{-1}(q) = TM(d)^q$ and $F^{-1}(q) = MZ^q$, where F^{-1} given the CDF of MZ and $TM(d)$.

4.2.5. *Reserved storage requirement constraint.*

$$Pr = \{M_{rtk_1} \geq RSR(r)^{\pm}, \quad t; \quad k_1, \ldots, k_1^t; \quad r = 1,2\} \geq 1-q, \tag{6k}$$

$$Pr = \{TT_{dtk_1} \geq RST(d)^{\pm}, \quad t; \quad k_1, \ldots, k_1^t; \quad d = 1,2\} \geq 1-q, \tag{6l}$$

$$Pr = \{MM_{k_1k_2} \geq RSZ^{\pm}, \quad t; \quad k_1, \ldots, k_1^t; \quad k_2, \ldots, k_2^t\} \geq 1-q, \tag{6m}$$

Constraints (6k), (6l), and (6m) indicate that, under all scenarios, the amount of water stored in each reservoir, treatment facility, and water resource will not lower the reserve level. Constraints (6k), (6l), and (6m) are altered to linear ones using the following formula:

$$M_{rtk_1} \geq (RSR(r)^{\pm})^q, \quad t; \quad k_1, \ldots, k_1^t; \quad r = 1,2, \tag{6n}$$

$$TT_{dtk_1} \geq (RST(d)^{\pm})^q, \quad t; \quad k_1, \ldots, k_1^t; \quad d = 1,2, \tag{6o}$$

$$MM_{k_1k_2} \geq (RSZ^{\pm})^q, \quad t; \quad k_1, \ldots, k_1^t; \quad k_2, \ldots, k_2^t. \tag{6p}$$

These constraints are linear, and $F^{-1}(q) = RSR(r)^q$ and $F^{-1}(q) = RST(d)^q$, where F^{-1} given the CDF of MZ and $TM(d)$.

4.2.6. *Constraints on the maximum amount of water that can be released from dams.*

$$\sum_{r=1}^{2} X^{\pm}_{rdtk_1} UX_{rd} - Q^{\pm}_{rtk_1} \leq MR_{rt}, \quad \forall t, \tag{6q}$$

4.2.7. *Non-negative constraint.*

$$H_t \geq Y^{\pm}_{tk_1k_2} \geq 0, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad k_2, \ldots, k_2^t, \tag{6r}$$

A two-step approach can be used to solve the model. Two sub-models are solved for each objective, yielding upper and lower bounds. When the system objective is to be maximized, the sub-model corresponding to f^+ and SL^+ can be formulated in the first step. The next sub-model (f^-, SL^-) can be expressed based on the first sub-model. For the net benefit, the first sub-model is as follows:

$$\begin{aligned}
&\text{Max} f^+ = \textstyle\sum_{t=1}^{T} NB_t^+ H_t - \sum_{t=1}^{T} \sum_{k_1=1}^{k_1^t} \sum_{k_2=1}^{k_2^t} p_{tk_1} p_{tk_2} PE_t^- Y^-_{tk_1k_2}, \\
&M_{r(t+1)k_1} = M_{rtk_1} + Q^+_{rtk_1} - \textstyle\sum_{d=1}^{2} X^-_{rdtk_1} UX_{rd} - E_r^-, \\
&TT_{d(t+1)k_1} = TT_{dtk_1} + \textstyle\sum_{d=1}^{2} X^+_{rdtk_1} UX_{rd} - \sum_{d=1}^{2} L^-_{dtk_1}, \\
&MM_{(t+1)k_1k_2} = MM_{tk_1K_2} + C^+_{tk_2} - \textstyle\sum_{d=1}^{2} T^-_{tk_1k_2} + \sum_{d=1}^{2} L^+_{dtk_1},
\end{aligned}$$

$$\begin{aligned}
&H_t \geq Y^-_{tk_1k_2} \geq 0, \quad \forall t; \quad \forall k_1, \ldots, k_1^t, \quad \forall k_2, \ldots, k_2^t, \\
&\sum_{r=1}^{2} X^+_{rdtk_1} UX_{rd} - Q^-_{rtk_1} \leq MR_{rt}, \quad \forall t,
\end{aligned}$$

$$M_{rtk_1} \geq (RSR(r)^-)^q, \quad \forall t; \quad \forall k_1, \ldots\ldots, k_1^t; \quad \forall r = 1,2 \tag{7a}$$
$$TT_{dtk_1} \geq (RST(d)^-)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall d = 1,2 \tag{7b}$$
$$MM_{k_1k_2} \geq (RSZ^-)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall k_2, \ldots, k_2^t; \tag{7c}$$
$$H_t^+ Y_{tk_1k_2}^- \leq T_{tk_1k_2}^+, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall k_2, \ldots, k_2^t, \tag{7d}$$
$$De_{t\min} \leq H_t \leq De_{t\max}, \quad \forall t, \tag{7e}$$
$$De_{t\min} \leq T_{tk_1k_2}^+ \leq De_{t\max}, \quad \forall t, \tag{7f}$$
$$TT_{dtk_1} \leq (TM(d)^+)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall d = 1,2, \tag{7g}$$
$$MM_{k_1k_2} \leq (MZ^+)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall k_2, \ldots, k_2^t; \tag{7h}$$

where H_t and $Y_{tk_1k_2}^-$, the solutions of Model (7), are decision variables. The second sub-model for f^- can then be written as:

$$\begin{aligned}
&\text{Max} f^- = \textstyle\sum_{t=1}^{T} NB_t^- H_t - \sum_{t=1}^{T} \sum_{k_1=1}^{k_1^t} \sum_{k_2=1}^{k_2^t} p_{tk_1} p_{tk_2} PE_t^+ Y_{tk_1k_2}^+ \\
&\text{s.t} \\
&M_{r(t+1)k_1} = M_{rtk_1} + Q_{rtk_1}^- - \textstyle\sum_{d=1}^{2} X_{rdtk_1}^+ UX_{rd} - E_r^+ \\
&TT_{d(t+1)k_1} = TT_{dtk_1} + \textstyle\sum_{d=1}^{2} X_{rdtk_1}^- UX_{rd} - \sum_{d=1}^{2} L_{dstk_1}^+ UL_{ds}, \\
&H_t \geq Y_{tk_1k_2}^+ \geq 0, \quad \forall t; \quad \forall k_1, \ldots, k_1^t, \quad \forall k_2, \ldots, k_2^t
\end{aligned}$$

$$\sum_{r=1}^{2} X_{rdtk_1}^- UX_{rd} - Q_{rtk_1}^+ \leq MR_{rt}, \quad \forall t,$$
$$M_{rtk_1} \geq (RSR(r)^+)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall r = 1,2, \tag{8a}$$
$$TT_{dtk_1} \geq (RST(d)^+)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall d = 1,2, \tag{8b}$$
$$MM_{k_1k_2} \geq (RSZ^+)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall k_2, \ldots, k_2^t; \tag{8c}$$

$$H_t - Y_{tk_1k_2}^+ \leq T_{tk_1k_2}^+, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall k_2, \ldots, k_2^t, \tag{8d}$$
$$De_{t\min} \leq H_t \leq De_{t\max}, \quad \forall t, \tag{8e}$$
$$De_{t\min} \leq T_{tk_1k_2}^- \leq De_{t\max}, \quad \forall t, \tag{8f}$$
$$TT_{dtk_1} \leq (TM(d)^-)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall d = 1,2, \tag{8g}$$
$$MM_{k_1k_2} \leq (MZ^-)^q, \quad \forall t; \quad \forall k_1, \ldots, k_1^t; \quad \forall k_2, \ldots, k_2^t; \tag{8h}$$

where $Y_{tk_1k_2}^+$ are decision variables. Thus, for the MSISP model, we have the following solutions:

$$Y_{tk_1k_2\text{opt}}^{\pm} = [Y_{tk_1k_2\text{opt}}^-, Y_{tk_1k_2\text{opt}}^+], \tag{9a}$$
$$f_{\text{opt}}^{\pm} = [f_{\text{opt}}^-, f_{\text{opt}}^+]. \tag{9b}$$

For the other objective function, social satisfaction, two sub-models are obtained. The upper and lower objective functions are

$$SL^{\pm}_{\text{opt}} = [SL^{-}_{\text{opt}}, SL^{+}_{\text{opt}}]. \tag{9c}$$

The optimal water-allocation scheme is:

$$A^{\pm}_{tk_1k_2\text{opt}} = H_{t\text{opt}} - Y^{\pm}_{tk_1k_{2\text{opt}}}, \quad \forall t; \quad \forall k_1, \ldots, k_1'; \quad k_2, \ldots, k_2'. \tag{9d}$$

The optimal levels for the single objective models are obtained to optimize each objective. The best and the worst values of each objective function need to be calculated. Then, the multi-objective model is solved by the compromise programming technique. A negative function of each function is optimized to calculate the worst values. We obtain an upper and lower bound for each element.

5. Results

5.1. The HPWN data. The information regarding water distribution supplies is presented in Tables 1 and 2. The economic and technical data are presented in Table 1.

TABLE 1. Economic and technical data.

	Planning period		
	$t=1$	$t=2$	$t=3$
Low flow stream1 (probability 0.15)($10^6 m^3$)	[12.20, 14.80]	[09.40, 11.60]	[11.71, 12.12]
Medium flow stream1 (probability 0.15)($10^6 m^3$)	[16.30, 19.51]	[16.20, 17.80]	[15.07, 16.27]
High flow stream1 (probability 0.15)($10^6 m^3$)	[30.41, 40.25]	[25.20, 33.04]	[28.76, 32.03]
Low flow stream2 (probability 0.15)($10^6 m^3$)	[06.50, 08.31]	[03.61, 04.02]	[7.38, 09.39]
Medium flow stream2 (probability 0.15)($10^6 m^3$)	[08.25, 09.33]	[06.82, 07.21]	[09.12, 10.91]
High flow stream2 (probability 0.15)($10^6 m^3$)	[14.68, 18.86]	[11.32, 15.03]	[14.80, 17.62]
Water allocation demand ($10^6 m^3$)	[27.31, 43]	[31.30, 48.14]	[32.83, 50.75]
Net benefit when water demand is satisfied	[218.61, 235.44]	[265.84, 285.86]	[307.82, 331.05]
Penalty when water is not delivered	[129.14, 142.11]	[146.16, 168.97]	[157.93, 179.90]

TABLE 2. Probabilistic data of the reserves and capacities.

	$q = 0.01$	$q = 0.05$	$q = 0.10$
Capacity of the reservoir $10^6 m^3$	$[67.85, 69.8]$	$[71.65, 74.4]$	$[73.65, 76.6]$
Capacity of treatment 1 $10^6 m^3$	$[7.06, 17.06]$	$[10, 20.08]$	$[11.4, 21.68]$
Capacity of treatment 2 $10^6 m^3$	$[14.03, 31.06]$	$[17.83, 35.36]$	$[19.83, 37.56]$
Reserve water for reservoir $10^6 m^3$	$[41.9, 48]$	$[38.7, 42.4]$	$[37.1, 40.60]$
Reserve water for dam 1 $10^6 m^3$	$[10, 13]$	$[8.2, 8.5]$	$[5.80, 10.00]$
Reserve water for dam 2 $10^6 m^3$	$[3.5, 5.3]$	$[1.7, 1.90]$	$[1.50, 3.00]$
Reserve water for treatment 1 $10^6 m^3$	$[4.43, 5.44]$	$[1.97, 2.68]$	$[0.77, 1.38]$
Reserve water for treatment 2 $10^6 m^3$	$[7.83, 13.73]$	$[4.63, 10.03]$	$[3.03, 8.13]$

Table 2 shows the reserves and capacities of the reservoir, and the treatments and regulations of the reserve water. The weights of the two objectives were calculated using the AHP technique as $w_1 = 0.3$ and $w_2 = 0.7$ and the level of satisfaction of water resources was determined as $Sw_1 = 0.44,\ Sw_2 = 0.26,$ and $Sw_3 = 0.18$. The concerns of the decision maker about maximum deviance from ideal values is reflected by parameter p. The two-objective model is solved by compromise programming. The best and worst values of each function are required for this technique. We used the Lingo software to code the IDMSISP model. In this study, the scenario trees were created by probabilistic random variables. For instance, a three-year period scenario tree can be constructed by stream 1; then, with the structure 1-3-3-3, a three-period (four-stage) scenario tree can be built. Consequently, many scenarios can produce with stream 2 and different joint probabilities. The optimal water allocation plans are shown in Figs. 2 and 3 for $q = 0.01,\ 0.1$, and 0.5. Figures 4 and 5 show the amount of shortage water for $q = 0.01,\ 0.1$, and 0.5. They show differences between the promised water allocation goal and the probabilistic shortage with an associated probability level under a given stream condition.

The results showed that the increase in q level may result in a reduction in the storage capacities. An increased storage capacity would enable reservoirs to hold more excess water when stream flows are strong in one period, resulting in fewer shortages in the next period. Reduced reserve requirements, conversely, would allow for less water to be held in reservoirs while flows are low over the planning horizon. As as the q level rises, there will be fewer water shortages and more water allocation. The results for system benefit and

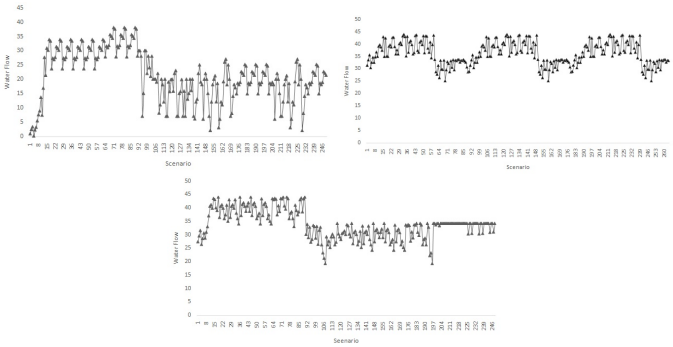

FIGURE 2. Water flow (upper) $q = 0.01$, $q = 0.1$, and $q = 0.5$ from left to right, respectively.

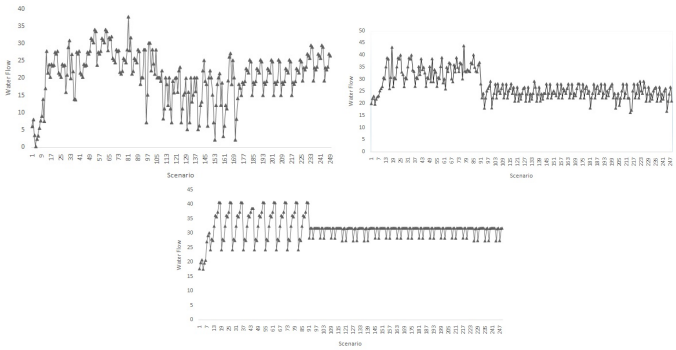

FIGURE 3. Water flow (lower) $q = 0.1$, $q = 0.01$, and $q = 0.5$ from left to right, respectively.

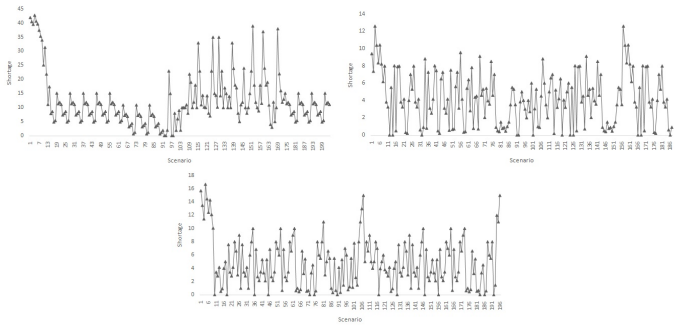

FIGURE 4. Water shortage (upper) $q = 0.1$, $q = 0.01$, and $q = 0.5$ from left to right, respectively.

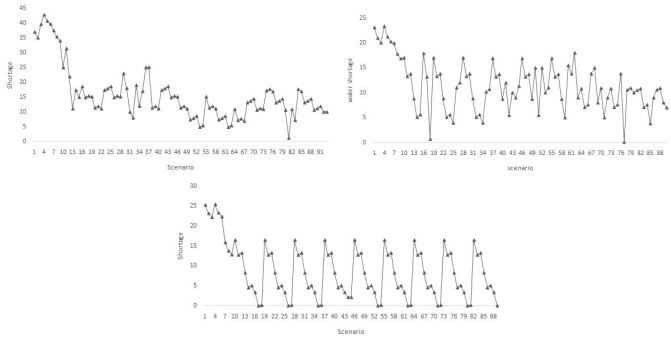

FIGURE 5. Water shortage (lower) $q = 0.1$, $q = 0.01$, and $q = 0.5$ from left to right, respectively.

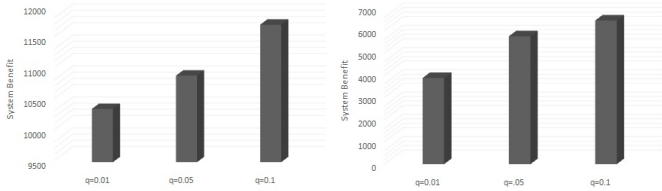

FIGURE 6. System benefit upper and lower bound from left to right respectively.

FIGURE 7. Social satisfaction upper and lower bound from left to right respectively.

social satisfaction under different q levels are presented in Figures 6 and 7. The results for both system benefit $f_{opt}^{\pm}$ and social satisfaction $SL_{opt}^{\pm}$ show that a lower q level results in lower social satisfaction; vice versa, a higher q level results in higher social satisfaction and system benefit. A higher q level is higher risk and sacrifices the safety of the system and the environmental restriction. Hence, there is a positive relationship between both social satisfaction and water allocation, and the safety of the system and the environmental restriction.

6. Notations and definitions

In implementing the model, certain parameters and decision variables were used. The notations and definitions used in our model are shown in Table 3.

TABLE 3. Notations and definitions.

Parameters	
M_{rtk_1}	Storage level in resource r in period t under scenario k_1 (m^3).
MR_{rt}	The maximum amount of water that can be released from resource r in period t.
$MM_{tk_1k_2}$	Inventory of reservoir under scenarios k_1 and k_2 in period t.
$Q^{\pm}_{rtk_1}$	Random inflow into stream 1 in period t under scenario k_1 that enters the dams.
$X^{\pm}_{rdtk_1}$	The amount of water moved from water resource r to treatment d in period t under scenario k_1.
UX_{rd}	Introduces the allowed paths from resource r to treatment d, which is a $0-1$ value.
$E^{\pm}_r$	Loss due to evaporation of resource r in period t, (m^3).
TT_{dtk_1}	Inventory of treatment d under scenarios k_1 and k_2 in period t.
$L^{\pm}_{dtk_1}$	The amount of water released from treatment d to reservoir under scenarios k_1 in period t.
$C^{\pm}_{tk_2}$	The amount of water released from wells to reservoir under scenarios k_2 in period t.
$T^{\pm}_{tk_1k_2}$	The volume of water pumped out of the reservoir to the consuming zone under scenarios k_1 and k_2 in period t.
$H^{\pm}_t$	The consuming zone's water allocation target in period t.
$Y^{\pm}_{tk_1k_2}$	Shortage level under scenarios k_1 and k_2 in period t.
De_{tmin}	The consuming zone's minimum water demand over period $t(m^3)$.
De_{tmax}	The consuming zone's maximum water demand over period $t(m^3)$.
$TM(d)$	Reserved storage level for treatment d.
$MZ^{\pm}$	Reserved storage level for reservoir.
$RSR(r)^{\pm}$	Capacity of storage for resource $r(m^3)$.
$RST(d)^{\pm}$	Capacity of storage for treatment $d(m^3)$.
$RSZ^{\pm}$	Capacity of storage for reservoir (m^3).
SW	The social satisfaction degree computed for each water resource.
k^t_1	Potential scenarios of stream 1 over period t.
k^t_2	Potential scenarios of stream 2 over period t
$NB^{\pm}_t$	Net benefit of per unit of water distributed over the time period t.
$f^{\pm}$	The net benefit of the system over the planning horizon \$.
t	The time period.
p_{tk_1}	Probability of scenario k_1 occurring for a stream in period t.
p_{tk_2}	Probability of scenario k_2 occurring for a stream in period t.

7. Conclusions

In this study, an IDMSISP was developed for water-resource allocation under uncertainty and was applied for a water distribution network. The following aspects were involved: stochastic programming, chance-constrain programming, interval programming and compromise programming. In this model, system uncertainties are dynamically considered through decision processes. The achieved outcomes showed that the proposed model can be applied as an effective mechanism for integrated urban water management. This model will

help in the optimal allocation of water supplies to consumption zones for water distribution with optimal time planning.

Acknowledgements

This work was started during a research stay of S. Sharifi and M. Khademi as visiting researchers at the "Università Mediterranea di Reggio Calabria", and of Dr. M. Salimi as a visiting professor. The research was partially supported by the "Dipartimento di Giurisprudenza, Economia e Scienze Umane" through the "Piano Dipartimenti Universitari di Eccellenza L. 232/2016".

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Manuscript received 26 May 2021; communicated 28 May 2021; published online 4 March 2022